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# Driving time, productivity, and the Fundamental Law of Road Congestion

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## Abstract

Road congestion is a negative externality associated to automobile use and can negatively affect drivers' utility in several directions, such as delay time and wasted fuel, but also can have a negative impact on aggregate productivity. This paper develops a Dynamic Stochastic General Equilibrium model to study the interactions between roads, traffic, congestion and productivity over the business cycle. In our model households receive services from vehicles, depending on the flow kilometers driving and on the stock of cars, and they do not take into account their own impact on congestion, which also depends on the stock of roads. Following a positive aggregate productivity shock, traffic density and congestion tend to rise, so dampening its positive effects on aggregate activity. The model is then used to study the process behind the so-called "Fundamental Law of Highways Congestion", which states that an increase in the stock of roads produces a traffic density rise of same proportion, thus leaving congestion unaffected in the long run. Our model economy reproduce a rise in output in response to a road capacity expansion and predicts a traffic flow elasticity of 0.172, as a direct consequence of the positive impact of the higher stock of roads on economic activity. Finally, we derive a Pigouvian tax schedule that internalizes the social costs of congestion. (*JEL* E32, R41, R42, R48)

## Keywords:

Road traffic, Congestion, The Fundamental Law of Road Congestion, Dynamic General Equilibrium model

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## 1. Introduction

Road congestion is a negative externality associated to automobile use and it can negatively affect welfare in several directions, including delay time and wasted fuel. According to a recent report by INRIX-CEBR (2015), people in Europe and the U.S. waste, on average, 111 hours annually in gridlock. As of 2013, traffic congestion costs for the U.S. are estimated to be 124,2 billion dollars. For European countries, estimated congestion costs are also large: 20.5 billion dollars for the U.K., 22.5 for France, and 33.4 for Germany. Just in Los Angeles, the estimate

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congestion cost is 23.2 billions dollars (INRIX-CEBR, 2015). In terms of annual hours wasted by passenger in 2013 were 68 in the U.S., 123,9 in the U.K., 118 in Germany and 135.8 in France. Durantón and Turner (2011) pointed out that in the year 2001, an average American household spent 161 person-minutes per day in a passenger vehicle. Similar conclusions are reported by Parry, Heine, Lis and Li (2014), who studied road traffic in several cities across the world. As of 1995, using the Millennium Cities Database, they estimate that the average delay time was 0.0058 hours per km. for 15 US cities. They also reported an average of 18.4 thousand kilometers driven. Multiplying these two figures, we obtain a total delayed hours per year of 107.3, a figure very similar to the estimated value in INRIX-CEBR (2015). Schrank and Lomax (2005) estimated that urban road congestion causes 3.7 billions hours of delay a year worldwide.

Congestion can be viewed as a function of traffic density relative to road capacity. When an additional vehicle incorporates in a road, from a certain level of traffic density, congestion increases as a higher proportion of road capacity is used, and thus, average speed decreases. When we add vehicles to a road, the distance between vehicles decreases and hence also decreases average speed, increasing travel time. If the number of vehicles in a particular road is low enough, the interactions among road users become smaller, resulting in a travel speed equals to the so-called "free flow vehicle speed" and no congestion.<sup>1</sup> However, as the number of drivers increases, interactions among them also increase, giving place to a negative externality in the form of higher driving time. Congestion costs have various consequences: time delayed, extra fuel consumption, higher vehicle operating costs and emissions, and a likely increase of crashes.<sup>2</sup> Congestion costs heavily depend on the value of travel time, which is related to wages for each country.

Road congestion can be apparently solved by investing in new road capacity given traffic density. An early contribution, however, Downs (1962), conjectured that as road capacity increases, traffic also increases in the same proportion, so as to keep congestion constant in the long term. This phenomenon was called the "Fundamental Law of Road Congestion". This law implies that building new roads does not help reduce congestion, as more traffic is generated in response to the new capacity. This "Fundamental Law or Road Congestion" is connected to a number of paradoxes related to road traffic, such as the Downs-Thomson paradox (Downs, 1962; Thomson, 1977), the Braess paradox (Braess *et al.*, 2005), and with the so-called Marchetti's constant (Marchetti, 1994).<sup>3</sup> Jorgensen (1947) was the first to estimate empirically the response of traffic

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<sup>1</sup>Free flow vehicle speed represents the speed at which drivers choose to travel without influence by other road users.

<sup>2</sup>The relationship between congestion and fuel consumption is not clear. Several authors pay attention on the fact that congestion involves higher consumption of fuel, since fuel efficiency decreases. For instance, Schrank, Lomax and Eisele (2011) estimated that additional fuel consumption from congestion is only about 5% of total cost, accounting the remaining 95% for the cost of additional time lost. However, other authors, as Greenwood and Bennett (1996), and Small and Gómez-Ibáñez (1998), pointed out that the relationship between slower traffic speed and fuel consumption is difficult to specify correctly.

<sup>3</sup>The Downs-Thomson paradox states that an improvement in the stock of roads could increase congestion by causing public transport users to shift to driving. The Braess paradox states that a reduction of road capacity could improve traffic flows and reduce congestion. Finally, the Marchetti's constant states the the average travel time per person each day is about one hour. An improvement in roads maintains constant the travel time thus increasing the distance and traffic.

to road capacity expansions, confirming only partially the fundamental law of congestion.<sup>4</sup> Following Litman (2016) generated traffic in response to road capacity expansion comes from two sources: Diverted traffic (trips shifted in time, route and destination, what Downs, 1992, call the *Triple Convergence*), and induced traffic (shifts from other modes, longer trips and new trips). Based on that fundamental law it results clear that generated traffic reduces benefits of road capacity expansion over congestion.

The Fundamental Law of Traffic Congestion and its related paradoxes raises an important policy recommendation: road congestion cannot be relieved by building new roads, whose construction is expensive. In this sense, the elasticity of traffic with respect to road enlargements is a central question in the debate about new road construction. A large body of empirical literature has explored the relationship between road capacity, measured either by lane-kilometers or as travel time, and traffic, measured as vehicle-kilometer traveled (VKT). However, estimated generated traffic associated to road improvements are very imprecise and estimated elasticity goes from 0 to 100 percent. In general, the majority of empirical estimations only support partially the fundamental law of highways congestion. Hansen (1995), for instance, reports values for the elasticity ranging from 0 to 0.3 one year after construction, and values from 0.2 to 0.8 four years later, using data from the California Metropolitan Areas. He finds, by contrast, that only a small fraction in the rise in VKT can be associated to the increase in lane-miles. Similarly, Cervero (2003), also using data from the California freeway, estimates a long-term elasticity of 0.64. Although estimated elasticity is always positive, they are usually below one.<sup>5</sup> Nevertheless, a recent article by Duranton and Turner (2011) estimated the elasticity of vehicle kilometers traveled with respect to lane kilometers, obtaining a value close to one for a number of alternative specifications. This is an empirical evidence in favor of the “fundamental law of highways congestion” as proposed by Downs (1962). Moreover, they suggest that the fundamental law can be extended to a broad class of major urban roads, resulting in a general “fundamental law of road congestion”. Based on these results, Duranton and Turner (2011) conclude that road capacity expansion is not an appropriate policy to reduce traffic congestion, and that policy interventions aimed at pricing congestion, are instead the correct instrument to internalize social costs from congestion. These policy measures include tolls, entry restrictions, or Pigouvian taxation. An estimated elasticity even larger than one is obtained by Hsu and Zhang (2014) for the case of Japan.

Road traffic and congestion problems have been studied in the literature using alternative models. Examples include those of Vickrey (1969), Henderson (1974, 1977, 1981), Newell (1987), Arnott, De Palma and Lindsey (1990, 1993, 1994), among many others. These models focus on microeconomic aspects and fail in taking into account the behavior of the different agents in the economy at an aggregated level. Little is known about the interactions between road traffic and the business cycle. This paper builds a dynamic stochastic general equilibrium (DSGE) model to

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<sup>4</sup>Jorgensen (1947) estimated the induced traffic from the construction of the Merritt and Wilbut Cross parkways in the corridor between New York city and New Haven, Connecticut. He estimated the relationship between traffic and gasoline sales in Connecticut before the construction of the parkways. Then, he used information on the growth in gasoline sales after the construction to estimate the growth in traffic, obtaining between 20 and 25 percent more traffic that would have been expected.

<sup>5</sup>See Cervero and Hansen (2002), and Cervero (2002) for surveys of the empirical literature, most of the works reporting elasticity estimates below one.

study traffic and congestion over the business cycle. Automobile traveling activities are included in households' utility function in two ways. First, we assume that households receive services from driving cars. Second, total available time can be allocated into leisure, working activities and driving, where driving has a complementarity relationship with working activities and it represents a disutility. Somewhat related to our model is the one by Wei (2013), who develops a DSGE model to study fuel efficiency. In our model, traffic is pro-cyclical as it is related to the economic activity given its complementarity with labor. This implies that congestion will be also pro-cyclical given the stock of lane kilometers. In fact, road congestion has decreased dramatically during the last downturn starting in 2008. On the other hand, different studies show that the stock of roads have increased at a lower rate than traffic has. For example, Parry, Walls and Harrington (2007) pointed out that between 1980 and 2003, urban vehicles miles traveled in the US increased by 111%, whereas road lane-mile capacity only increased by 51%, implying that annual urban congestion delays increased from 16 to 47 hours per driver. A result we find is that the effects of a positive aggregate shock are dampened as the level of congestion increases. Moreover, we find a negative effect on hours worked (and leisure) as additional time is wasted in driving. Therefore, congestion can be viewed not only as a negative externality related to road traffic affecting welfare but a factor slowing down business activity. The model economy is used to study the effects of two additional shocks: A fuel price shock and a cars price shock. Two other shocks are studies using our model economy. First, it is found that a fuel price shock can have positive effect on economic activity in an environment of congestion. The negative effects of this shock on driving operative costs are compensate by the reduction in the level of congestion. Second, a cars price shock reduces the stock of cars but increases their use intensity thus provoking a rise in congestion.

Our model helps to analyze the conditions under which the "fundamental law" is confirmed. One important result from our model is that increasing road stock also rises VKT and hence, generated traffic is endogenously produced. The rise in VKT is about 0.172 of the rise in the stock of road, and hence, the level of congestion is reduced as road investment is increased, a response consistent with most of the empirical literature. The transmission mechanism is as follows. As drivers perceive the immediate benefits from extra reductions in delay time, traffic density increases due to a rise in the kilometers driven. In a later moment, this situation induces additional advantages to purchase new cars, so that the stock of cars increases, which further soars traffic density. There is an additional channel. Road is an input in the aggregate production function, which complements to capital and labor. Therefore an expansion in road capacity motivates economic activity. Fernald (1999) finds that vehicle-intensive industries benefit more from road-building, arguing that public investment may account for a substantial share of the slowdown in productivity growth after 1973, when congestion became important. Fernald (1999) reports evidence that the road stock per capita remained almost constant during the period 1973-1994 while traffic grew steadily. Based on these results, our recommendation differs from that of Duranton and Turner (2011), as our model hinges on these features that makes road investment a tool for boosting economic growth.

Finally, the model is thought to represent the congestion of urban areas. The central planner problem will be referred as the city mayor's problem: an agent that takes decision of consumption, road investment, hours worked, driving, while internalizing the social costs of congestion. This way, using the optimal allocations derived from the mayor's problem, we propose a Pigouvian tax schedule that internalizes congestion costs. We studies two taxes: a fuel price tax and a car sales

tax. We find that a car sales tax does not help to internalizing the cost of congestion and that it can be done through a fuel tax.

The remaining of the paper is structured as follows. In Section 2 we present a DSGE model in which road traffic is added to a otherwise standard macroeconomic model. The calibration exercise for the U.S. economy is presented in Section 3. Section 4 studies the dynamic properties of the model in the short-run, presenting the impulse-response function of the relevant variables to several shocks: An aggregate productivity shock, a fuel price shock, and a cars price shock. Section 5 uses the model to theoretically substantiate the Fundamental Law of Road Congestion, studying the response of the relevant variables a an increases in road capacity. Section 6 quantify the costs of congestion. Finally, Section 7 presents the main conclusions.

## 2. The model

In this section, we develop a dynamic stochastic general equilibrium (DSGE) model in which personal transport activities are included in household's utility function, road infrastructure in the production function, and road investment in the government (the city hall) budget constraint. Driving cars enters in the households utility function in two ways. First, households receives services from driving cars. Second, driving time is related to working time and labor is measured in efficient units where cars are considered as a working tools which complement labor hours. Public investment decisions are transformed in public capital (roads) stock. Finally, road infrastructure is considered as an additional input in the aggregate production function.

In the model described so far, lowercase letters will refer to individual variables, while uppercase letters refer to aggregate variables. This distinction is important given that individuals do not take into account the effects of their own driving decisions on aggregate traffic intensity.

### 2.1. Households

The representative household has a concave utility function which depends on consumption  $c_t$ , services from vehicles  $s_t$ , hours worked  $h_t$ , and driving time  $d_t$ , respectively. The utility function is assumed to have the following functional form:

$$u(c_t, s_t, h_t) = \ln c_t + \varphi_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(h_t + d_t)^{1+1/\nu}}{1+1/\nu}, \quad (1)$$

where  $\gamma > 0$  denotes degree of concavity in the utility from vehicle services.<sup>6</sup> The duet of parameters ( $\varphi_s > 0$ ,  $\varphi_h > 0$ ) denote the willingness to drive and the willingness to work, respectively, and where  $\nu$  is the Frisch elasticity of labor supply. Notice that, given the functional form assumed for the utility, working activities requires some additional non-paid time devoted to travel activities. Given that this paper is focused on congestion, leisure time while driving is excluded from the definition of driving time.

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<sup>6</sup>Similar utility functions to (1) are those used by Parry and Small (2005) and by Wei (2012). Parry and Small (2005) consider a static model where household's utility function depends on consumption, vehicle-miles of travel, time spent driving and leisure. A more related paper to our work is Wei (2012), who uses a logarithmic utility function for consumption, travel and leisure to study vehicle fuel efficiency.

Services from vehicles are assumed to depend on the flow kilometers driven,  $m_t$ , and on the current stock of cars,  $q_t$ :

$$s_t = q_t m_t^\phi, \quad (2)$$

where  $0 < \phi < 1$ , implying that using cars too intensely has diminishing returns i.e., it is better to use the fleet less intensively by having more cars.

Following Arnott, de Palma and Lindsey (1993), total driving time is the sum of free-speed driving time, which it is assumed to be fixed, plus congestion time, which depends on traffic density relative to road capacity. Notice that in our model, the value of travel time is defined endogenously. In the literature (mainly in Cost-Benefit Analyses), it is assumed that the opportunity cost of travel time is different from the opportunity cost of working time, although both are related to wages. Empirical literature uses a value of travel time half the market wage (see, for instance, Small and Verhoef, 2007). In general, the opportunity cost of working time is measured in lost wage. Here, the opportunity cost of travel time is defined in a somewhat different manner, as forgone wage and forgone leisure, as discretionary available time is splitted in three components: Leisure, working time, and driving time. For the sake of simplicity, the economy's total available amount of discretionary time is normalized to one, which we assume to be distributed between leisure, working activities and driving time:

$$leisure = 1 - h_t - d_t. \quad (3)$$

Lets  $M_t$  and  $Q_t$  denote aggregate values of kilometers driven and the aggregate stock of vehicles, respectively, and let  $K_{g,t}$  denote the stock of road available for drivers at time  $t$ . Traffic density is accounted by  $Q_t M_t$ . Road congestion is defined as traffic density relative to the stock of roads:

$$congestion = \frac{Q_t M_t}{K_{g,t}}. \quad (4)$$

In turn, we assume that time delay while driving exponentially depends on congestion:

$$d_t = \eta_0 \left( \frac{M_t Q_t}{K_{g,t}} \right)^{\eta_1}, \quad (5)$$

with  $\eta_0 > 0$ , and where  $\eta_1 > 0$  represents the elasticity of driving time with respect to congestion. This specification for driving time is analogous to those used by transportation engineers for travel delay (Parry *et al.*, 2014).<sup>7</sup> This way, delay time can be viewed as a negative externality from the congestion of public infrastructure. Under a decentralized framework, individuals' choices take road density  $M_t Q_t$  as given, that is, households do not consider their own use of car to affect traffic congestion.

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<sup>7</sup>This specification for driving time is similar to the one used by transportation engineers for travel delay, as shown by Parry *et al.* (2014) where a power function of traffic volume relative to road capacity is used. Small (1992), Small and Verhoef (2007). Parry and Small (2005) and Wei (2012) consider driving time as the inverse of the average travel speed times the aggregate miles of driving per capita, where the inverse of the average travel time is a function of the aggregate vehicle-miles of travel, as higher vehicle-miles of travel leads to more congested roads.

In our model, driving activities correspond to commuter trips. Time spent commuting is not leisure, and therefore, implies a disutility. On the other hand, time spent driving is not productive (although compulsory) and hence, no direct income is produced by this allocation of non-leisure time. Furthermore, this activity implies additional costs, such as fuel consumption and other operational costs related to vehicles' use. This is equivalent to the generalized cost (combination of both monetary and non-monetary costs) as defined by the transportation literature.

Following Fisher (2007), we allow a degree of complementarity between vehicles services,  $s_t$ , and the supply of labor, such that the amount of hours measured in efficiency units,  $\tilde{h}_t$ , are given by:

$$\tilde{h}_t = h_t^\theta s_t^{1-\theta}, \quad (6)$$

with  $\theta \in [0, 1]$ <sup>8</sup>. Such a complementarity between vehicle services and hours worked responds to the idea that cars can be viewed as work tools and are not mere consumption durable goods. In this respect, we follow Fisher (2007) and assume that the utilization of automobiles affects productivity and increases hours worked in efficiency units.

To render proper services, cars need to use resources that affect household's budget. We assume that these expenditures can be split into variable costs and fixed costs. Variable costs are those related to fuel and to maintenance and repairs. Fuel consumption is assumed to be proportional to total kilometers driven,  $m_t q_t$ . Let  $\omega_F > 0$  denote liters per kilometer (or gallons per mile in U.S. terminology), taken as constant for simplicity.<sup>9</sup> The lower  $\omega_F$ , the larger fuel efficiency is. Fuel can be purchased at a certain price in an oil station, including a tax per liter of fuel, and therefore, total expenditure on fuel can be defined as:

$$(p_t^F + \tau_t^F) \omega_F m_t q_t, \quad (7)$$

where  $p_t^F$  denotes the net price of fuel, and  $\tau_t^F$  represents the fuel tax which it is considered as a lump-sum tax. In a similar manner, vehicle maintenance and repairs are also assumed to be proportional to kilometers driven by a parameter  $\omega_{MR} > 0$ :

$$p_t^{MR} \omega_{MR} m_t q_t, \quad (8)$$

where  $p_t^{MR}$  denotes the price per unit of maintenance and repair services.

Denoting  $o_t$  the operating (variable) cost per kilometer, which includes fuel consumption plus maintenance and repairs costs, collected through expressions (7) and (8), we may write it in a compact notation:

$$o_t = (p_t^F + \tau_t^F) \omega_F + p_t^{MR} \omega_{MR}. \quad (9)$$

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<sup>8</sup>Standard RBC models suppose  $\theta = 1$ , implying that effective hours worked equal hours devoted to non-leisure activities. However, this case makes consumption of durables decrease in response to a positive shock to TFP, a prediction not supported by the data. Assuming, instead,  $\theta < 1$  helps the standard model to reconcile with the data (see Fischer, 2007).

<sup>9</sup>Aghion *et al.* (2012) show evidence that automotive industry tend to innovate relatively more in fuel efficiency under tax-adjusted fuel prices increases. Wei (2013) uses a vintage capital within a DSGE model to incorporate innovations in fuel economy. We opt to take fuel efficiency  $\omega_F$  as a parameter.



Operating costs are assumed to be proportional to driving distance. In practice, operating costs can also be affected by congestion when stop-and-go conditions occurs. Aggregating over the total variable utilization costs of vehicles gives

$$o_t m_t q_t. \quad (10)$$

Some other expenditures correspond to fixed costs. These expenditures, such as fees, tolls and car insurances, do not depend on their use, but solely on their property:

$$p_t^{TI} q_t, \quad (11)$$

where  $p_t^{TI}$  denotes an aggregate price for these fixed costs.

Denoting  $x_t$  as the amount of new cars purchases, vehicles  $q_t$  are accumulated according to the following geometric form:

$$q_{t+1} = (1 - \delta_q) q_t + x_t, \quad (12)$$

where  $0 < \delta_q < 1$  denotes cars' depreciation rate (including scrapped cars), and  $x_t$  represents units of brand new cars, which can be traded at a price  $p_t^X (1 + \tau_t^X)$ , with  $\tau_t^X$  being an indirect tax on the acquisition price.

Savings from households are accumulated in a capital asset  $k_t$ , which is employed by firms to produce goods, rendering  $R_t$  units of income per unit of capital  $k_t$ . This asset is accumulated according to:

$$k_{t+1} = (1 - \delta_k) k_t + i_t, \quad (13)$$

where  $i_t$  accounts for household's gross investment and  $\delta_k$  is the capital depreciation rate.

Finally, household's budget can be written as follows:

$$c_t + (1 + \tau_t^X) p_t^X x_t + i_t + (o_t m_t + p_t^{TI}) q_t = W_t \tilde{h}_t + R_t k_t + \pi_t + TR_t, \quad (14)$$

where  $(W_t, R_t)$  represent the hourly wage and the rental price of capital, respectively,  $\pi_t$  are firm profits, and  $TR_t$  are net lump-sum transfers.

## 2.2. Firms

We consider a representative profit maximizer firm that faces perfectly competitive markets of goods and factors. This firm produces a numeraire good  $Y_t$ , whose market price can be normalized to one. The firm hires capital and hours worked,  $(K_t, H_t)$ , and exploits a positive externality from the public stock of capital,  $K_{g,t}$ . Resources are transformed into the final good using the following CES technology:

$$Y_t = A_t \left[ \mu K_t^\rho + (1 - \mu) K_{g,t}^\rho \right]^{\alpha/\rho} \tilde{H}_t^{1-\alpha} \quad (15)$$

, where  $A_t$  represents the total factor productivity<sup>10</sup>,  $0 < \alpha < 1$  is the capital income share of output, and  $\mu \in (0, 1)$  the weight on private capital relative to public road infrastructure.  $\rho < 1$

<sup>10</sup>It is assumed for  $A_t$  to follow a AR(1) stationary process such that:  $\ln A_t = (1 - \zeta_A) \ln \bar{A} + \zeta_A \ln A_{t-1} + \varepsilon_t^A$ , where  $\varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$  and  $|\zeta_A| < 1$ .

determines the elasticity of substitution between private capital and road infrastructure. The degree of complementarity between  $K_t$  and  $K_{g,t}$  increases as  $\rho$  tends to  $-\infty$ . The technology presents constant return to scale for  $(K_t, H_t, K_{g,t})$ . Note that profits are not zero in equilibrium, once we account by the non paid rents from using the public input  $K_{g,t}$ . These rents will be rebated to consumers via lump-sum transfers  $\pi_t$  in budget (14).

### 2.3. The city hall (the government)

This city hall is headed by a mayor that must take decisions about taxes and the construction of road infrastructure. Tax revenues come from an ad valorem tax on fuel consumption  $\tau_t^F$ , and a sale tax on new cars  $\tau_t^X$ , levied on the price of new vehicles:  $(1 + \tau_t^X) p_t^X$ . Fiscal revenues,  $FR_t$ , therefore amount to:

$$FR_t = \tau_t^F \omega_F m_t q_t + \tau_t^X p_t^X x_t. \quad (16)$$

The city hall uses these revenues to finance expenditure in highway investment  $I_{g,t}$ , and balances budget in every period:

$$FR_t = I_{g,t} + TR_t. \quad (17)$$

where  $TR_t \geq 0$  are lump-sum transfers rebated to the households. We define an exogenous rule for public road investment:

$$I_{g,t} = \xi FR_t \quad (18)$$

where  $\xi$ , is the proportion of fiscal revenues devoted to road investment.

The stock of public input  $K_{g,t}$  (roads) is accumulated according to

$$K_{g,t+1} = (1 - \delta_g) K_{g,t} + I_{g,t}, \quad (19)$$

with  $0 < \delta_g < 1$  being the road depreciation rate.

All in all, under the assumption that the set of prices  $(p_t^X, p_t^F, p_t^{MR}, p_t^{TI})$  are exogenously given in this economy, the particular feasibility constraint of our model can be expressed in the following terms:

$$C_t + I_t + I_{g,t} + p_t^X X_t + (p_t^F \omega_F + p_t^{MR} \omega_{MR}) M_t Q_t + p_t^{TI} Q_t = Y_t. \quad (20)$$

### 2.4. Competitive problem

In a competitive environment, the representative household maximizes the lifetime utility, subject to the budget constraint and the state equations for  $k_t$  and  $q_t$  :

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(c_t) + \varphi_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(h_t + d_t)^{1+1/\nu}}{1+1/\nu} \right), \quad (21)$$

with respect to  $(c_t, h_t, m_t, x_t, i_t, k_{t+1}, q_{t+1})$  and subject to

$$c_t + (1 + \tau_t^X) p_t^X x_t + i_t + (o_t m_t + p_t^{TI}) q_t = W_t \tilde{h}_t + R_t k_t + \pi_t + TR_t, \quad (22)$$

$$k_{t+1} = (1 - \delta_k) k_t + i_t, \quad (23)$$

$$q_{t+1} = (1 - \delta_q) q_t + x_t. \quad (24)$$

where  $d_t$  is given by (5), and  $s_t$  is given by (2), (see Appendix A).

Summarizing, the first order conditions can be expressed as follows. First, a dynamic first order condition that determines the decision of investment in capital asset  $k_t$ :

$$\frac{1}{c_t} = \beta \mathbb{E}_t \left[ \frac{1}{c_{t+1}} (1 - \delta_k + R_{t+1}) \right]. \quad (25)$$

Second, the model produces a static condition accounting for the leisure-consumption trade-off, given the real wage  $W_t$ . Note that this expression also incorporates the driving time  $d_t$ , as a by-product from the utilization of cars:

$$\varphi_h (h_t + d_t)^{1/\nu} = \theta \frac{s_t^{1-\theta} W_t}{h_t^{1-\theta} c_t}. \quad (26)$$

Third, the decision of driving (kilometers traveled) is determined in a static condition, given the current state of vehicles  $q_t$ :

$$\varphi_s \phi m_t^{\phi(1-\gamma)-1} q_t^{-\gamma} = \frac{o_t - \phi(1-\theta) W_t h_t^\theta m_t^{\phi(1-\theta)-1} q_t^{-\theta}}{c_t}. \quad (27)$$

When the operating cost,  $o_t$ , or one of the components therein, increases, the household reduces the driving distance.<sup>11</sup>

Finally, the purchase of brand new vehicles (cars investment) is determined by the following dynamic equilibrium condition:

$$\frac{(1 + \tau_t^x) p_t^x}{c_t} = \beta \mathbb{E}_t \left[ (1 - \delta_q) \frac{(1 + \tau_{t+1}^x) p_{t+1}^x}{c_{t+1}} + \right. \quad (28)$$

$$\left. + \varphi_s m_{t+1}^{\phi(1-\gamma)} q_{t+1}^{-\gamma} + \left( (1 - \theta) W_{t+1} \frac{h_{t+1}^\theta}{q_{t+1}^\theta} m_{t+1}^{\phi(1-\theta)} - o_{t+1} m_{t+1} - p_{t+1}^{TI} \right) \frac{1}{c_{t+1}} \right], \quad (29)$$

The two last terms in this Euler equation can be viewed as the pricing kernel of automobiles, that is, the net gain from owning a car:

$$\Phi_t \equiv \underbrace{\varphi_s m_t^{\phi(1-\gamma)} q_t^{-\gamma} + (1 - \theta) W_t \frac{h_t^\theta m_t^{\phi(1-\theta)}}{q_t^\theta} \frac{1}{c_t}}_{\text{Marginal gain}} - \underbrace{\left[ o_t m_t + p_t^{TI} \right] \frac{1}{c_t}}_{\text{Marginal cost}}, \quad (30)$$

i.e. the marginal gain from one additional automobile minus the marginal opportunity cost. Iterating forward on this expression, we reach the following pricing expression:

$$(1 + \tau_t^x) p_t^x \frac{1}{c_t} = \frac{1}{1 - \delta_q} \sum_{j=1}^{\infty} \beta^j (1 - \delta_q)^j \mathbb{E}_t \left[ \Phi_{t+j} \right]. \quad (31)$$

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<sup>11</sup>When  $\theta = 1$ , kilometers traveled in expression (27) reduces to:  $m_t = \left[ \frac{\varphi_s \phi c_t}{q_t^\theta o_t} \right]^{\frac{1}{1-\phi(1-\gamma)}}$ . Given the stock of vehicles  $q_t$ , the elasticity of kilometers driven with respect to the operating cost  $o_t$  is given by  $\epsilon_{oc}^{km} = \frac{1}{1-\phi(1-\gamma)}$ .

The problem of profit maximization takes the following structure:

$$\max_{(\bar{H}_t, K_t)} \left[ \pi_t = Y_t - W_t \bar{H}_t - R_t K_t \right], \quad (32)$$

This problem produces the following first order conditions:

$$W_t = (1 - \alpha) \frac{Y_t}{\bar{H}_t}. \quad (33)$$

$$R_t = \alpha \frac{Y_t}{K_t} \frac{\mu K_t^\rho}{\mu K_t^\rho + (1 - \mu) K_{g,t}^\rho}. \quad (34)$$

These two conditions state that firms will demand hours worked and capital so that the value of their marginal products will equate the hiring costs,  $W_t$  and  $R_t$ , respectively. Since  $K_{g,t}$  is supplied by the government, it is considered to be exogenously given for firms.

Given these two expressions, unpaid rents from the public input  $K_g$  can be expressed as

$$\pi_t = \alpha Y_t \frac{(1 - \mu) K_{g,t}^\rho}{\mu K_t^\rho + (1 - \mu) K_{g,t}^\rho} > 0. \quad (35)$$

## 2.5. Equilibrium

Let  $\zeta_t = (k_t, q_t)$  denote the vector of individual state variables. Given a government policy,  $\{\tau_t^F, \tau_t^X, I_{g,t}, TR_t\}$ , a competitive equilibrium is a set of decision rules,  $\{c(\zeta_t), x(\zeta_t), m(\zeta_t), h(\zeta_t), k(\zeta_{t+1})\}$ , aggregate choices,  $\{C(\zeta_t), X(\zeta_t), M(\zeta_t), H(\zeta_t), K(\zeta_{t+1})\}$ , prices for fuel  $p_t^F$ , new vehicles  $p_t^X$ , maintenance and repairs  $p_t^{MR}$ , and tolls and cars insurances  $p_t^{TI}$ , and factor prices  $W(\zeta_t)$  and  $R(\zeta_t)$ , such that

1. Given the government policy and factor prices, households' decisions are maximized (21), subject to the budget constraint (23), the state equations for capital (24), vehicle accumulation (12), and non-negativity constraints.
2. All factors (hours and capital) are hired at their marginal productivities: (33), (34).
3. The government satisfies its budget constraint (17) every period.
4. Markets clean: labor demand is equal to labor supply. Condition (13) holds for physical capital. The feasibility condition for the final good holds, (20).
5. The representative agent condition holds, i.e. aggregate choices coincide with individual ones when the latter is representative:

$$\begin{aligned} K(\zeta_{t+1}) &= k(\zeta_{t+1}), \\ Q(\zeta_{t+1}) &= q(\zeta_{t+1}), \\ H(\zeta_t) &= h(\zeta_t), \\ C(\zeta_t) &= c(\zeta_t), \\ X(\zeta_t) &= x(\zeta_t). \end{aligned}$$

### 3. Calibration

In order to examine the quantitative implications of our model, we need to assign a value to the different parameters. The set of (a total of 19) parameters of the model are the following:  $(\beta, \delta_k, \varphi_h, \nu, \theta, \varphi_s, \phi, \gamma, \omega_F, \omega_{MR}, \delta_q, \alpha, \rho, \delta_g, \eta_0, \eta_1, \bar{A}, \mu, \xi)$ . Additionally, the model includes two tax rates:  $(\tau^F, \tau^x)$ . four exogenous prices:  $(p^x, p^F, p^{MR}, p^{T'})$ , and the parameters associated with the stochastic processes assumed for the shocks:  $\zeta_A, \sigma_A, \zeta_F, \sigma_F, \zeta_X, \sigma_X$ .

We may categorize them mainly as follows: parameters associated with preference specifications, parameters related to technology, parameters that relate to travel time and car use, and tax rates. Some of them can be set directly without solving the model, whereas others require the calculation of the steady-state to set its value as a function of a number of fundamental targets. Table 1 summarizes the set of targets for which the model is calibrated. Table 2 summarizes the calibrated values determined *ex-ante* (externally), whereas Table 3 summarizes calibrated values internally.

#### 3.1. Model targets

Parameters of the model are calibrated to match a number of targets, which are summarize in table 1. Steady state output is normalized to one,  $Y^* = 1$ . Private gross investment over GDP in steady state ( $I^*/Y^*$ ) is fixed to be 0.1766. The fraction of hours worked over total available discretionary time (which it is also normalized to 1) is 0.3333, using data from BLS (Bureau of Labor Statistics) and assuming a total available discretionary time of six days by week times 52 weeks by year times 16 hours by day (we assume that 8 hours are needed for sleeping).  $D^*$  is obtained as the fraction of hours driving. Duranton and Turner (2011) pointed out that in 2001, an average American household spent 161 person-minutes per day in a passenger vehicle (2.68 hours), roughly 978 hours per year. NHTS (National Households Travel Survey) surveys give lower values for the year 2001, around 80 minutes spent driving a private vehicle. Dividing by 16 hours a day (our measure of total available discretionary time), the resulting fraction of total hours driving is  $2.68/16=0.1675$ , according to Duranton and Turner (2011) data. However, that figure includes not only driving related to working activities but also driving time related to leisure. By contrast, Wei (2013) calibrated driving time to be a 4% of household discretionary time. NHTS surveys find that around 27% of total VKT per household corresponds to travel to/from work, or to work related to business. Using Duranton and Turner (2011) and NHTS data, driving time related to working activities is about 0.7236 hours per day, i.e.:

$$D^* = \frac{0.7236}{16} = 0.0452 \quad (36)$$

Following Parry *et al.* (2014), delayed driving time is 107.3 hours per year, or:

$$Delay = \frac{107.3}{365 \times 16} = 0.0184 \quad (37)$$

This implies that in steady state, a total of  $1-0.3333-0.0452=0.6325$  fraction of time is leisure, defined as all other activities but working or working-related driving, and that approximately, about 41% of total driving time is due to congestion.

As of 1995, NHTS reports an estimate average of 1.33 workers per household and an average of 1.32 vehicles per worker. From these figures we obtain an absolute stock of vehicles per households of  $Q^* = 1.33 \times 1.34 = 1.7822$ . Parry *et al.* (2014) report an average of 18.5 thousand kilometers driven and hence we set  $M^* = 18.5$ . These figures implies a steady-state value for vehicle-kilometers traveled of  $VKT = M^*Q^* = 1.7822 \times 18.5 = 32.9707$ .

**Table 1: Model targets**

Target	Notation	Value	Source
Output (GDP)	$Y^*$	1.0000	Normalization
Fraction of hours worked	$H^*$	0.3333	BLS
Fraction of hours driving	$D^*$	0.0452	NHTS
Private gross investment over GDP	$I^*$	0.1766	BEA
New cars investment over GDP	$p^X X^*$	0.0291	BEA
Fuel expenditures over GDP	$F^*$	0.0201	BEA
Maintenance and repairs over GDP	$Z^*$	0.0162	BEA
Fixed costs (tolls and insurances) over GDP	$TI^*$	0.0070	BEA
Gross stock of passenger vehicles	$Q^*$	1.7822	NHTS
Number of kilometers driven (thousand)	$M^*$	18.500	Parry <i>et al.</i> (2014)
Public gross ROAD investment over GDP	$I_g^*$	0.0148	BEA

According to Eurostat database, which provides a detailed structure for transport expenditures per household; fuel expenditure accounts for 3.78% while maintenance and repairs expenditures account for 2.44% of consumption expenditure in 1999. The share of cars insurance expenditures, only available for 2005 and 2010, average value is of 1.5% of consumption expenditures. For the U.S., we use data from the BEA. Average fuel consumption over GDP is 0.0201, whereas maintenance and repairs represents a 1.62% of GDP. Finally, fixed costs including tolls and insurances over GDP is 0.007. Therefore, for the steady state conditions we choose the following targets:

$$\frac{F^*}{Y^*} = (p_F + \tau^F)\omega_F M^* Q^* = 0.0201, \quad (38)$$

$$\frac{Z^*}{Y^*} = p_{MR}\omega_{MR}M^*Q^* = 0.0162, \quad (39)$$

$$\frac{TI^*}{Y^*} = p_{TI}Q^* = 0.0070, \quad (40)$$

These figures implies that about 7.24 percent of GDP is expending in goods associated with cars use. Since  $p_{MR}$  is normalized to 1, we can state that  $\omega_{MR}$  equals 0.0162 and that  $p_{TI}$  equals 0.007. Moreover, since  $p_F$  is set equal to 3.150 dollars per gallon, we obtain a value for  $\omega_F$  equal to 0.0064. Also from the BEA we obtain that the fraction of new cars investment over GDP is 0.0291.

Finally, According to the International Transport Forum, investment in inland transport investment has remained constant, at about 0.8% of GDP (excluding Japan). Nevertheless, using data from BEA, average road government investment in the U.S. is 1.48% of GDP, that is  $I_g^*/Y^* = 0.0148$ .

**Table 2: Ex-ante calibrated parameters**

Category	Notation	Parameter definition	Value	Source
Preferences	$\nu$	Frisch elasticity of labor supply	0.7200	Heathcote <i>et al.</i> (2010)
	$\theta$	Fischer complementarity hours-cars	0.9280	Fischer (2007)
	$\beta$	Discount factor	0.9902	Real interest rate of 4%
Technology	$\alpha$	Capital income share	0.3300	Gollin (2002)
	$\rho$	Complementarity capital-to-road	-0.5000	Ad hoc
	$\delta_k$	Capital depreciation rate	0.0150	BEA
	$\delta_g$	Road depreciation rate	0.0150	BEA
Travel and car use	$\eta_1$	Elasticity of driving time wrt congestion	2.5000	Parry <i>et al.</i> (2014)
	$\delta_q$	Cars depreciation rate	0.0208	BEA
Exogenous prices	$p^F$	Price of fuel	3.1500	EIA
	$p^{MR}$	Price of maintenance and repairs	1.0000	BEA
Tax rates	$\tau^F$	Fuel (ad valorem) tax	0.4950	EIA
	$\tau^x$	VAT tax pm cars' purchase price	0.0670	BEA
Exogenous shocks	$\varsigma_A$	Persistence of TFP shocks	0.9517	Estimation
	$\sigma_A$	Standard deviation of TFP shocks	0.0075	Estimation
	$\varsigma_F$	Persistence of fuel price shocks	0.9590	Estimation
	$\sigma_F$	Standard deviation of fuel price shocks	0.0676	Estimation
	$\varsigma_X$	Persistence of cars price shocks	0.9746	Estimation
	$\sigma_X$	Standard deviation of cars price shocks	0.0050	Estimation

### 3.2. Parameters calibrated externally

*Preferences* ( $\nu, \theta, \beta$ ): First, the calibrated value for the Frisch elasticity,  $\nu$ , is set equal to 0.72 according to Heathcote *et al.* (2010). Chetty *et al.* (2011) obtained a similar value using micro data and considering a married couple as the notion of household. This value is consistent with the micro evidence since we assume stable individuals, see Dyrda *et al.* (2012). The parameter representing the complementarity between hours and cars,  $\theta$  is set equal to 0.928 based on the correlation between national income and new cars investment data from the Bureau of Economic Analysis, as shown by Fischer (2007). From the steady state conditions from the model we obtain that:

$$\beta = \frac{1}{1 - \delta_k + R^*}, \quad (41)$$

The discount factor,  $\beta$  takes a value of 0.9902, a value standard in the literature when considering quarterly data. This value is selected to match a value for the real interest rate net of depreciation of 4% annually.

*Technology* ( $\alpha, \rho, \delta_k, \delta_g$ ): Capital income's share of output,  $\alpha$ , is set equal to 0.35. This parameter can be measured with data from the National Accounts. This share is consistent with those provided by Gollin (2002), who estimated that the capital income share should be within the [0.2, 0.35] interval in a wide set of countries under consideration. For the U.S. he reported a range from 0.257 to 0.396. Roads are used to commute and transport things. In the literature, it is standard to assume a Cobb-Douglas production function with both public and private capital. Examples

are Finn (1993), Guo and Lansing (1997), and Cassou and Lansing (1998), as they consider both types of capital assets as homogeneous, and hence, the elasticity of substitution between public and private capital is unitary. Nevertheless, here we relax that assumption, as we consider a complementarity relationship between private capital and the stock of roads. Thus,  $\rho$  is set equal to -0.5 as a benchmark to reflect some complementarity between capital and roads. Roads depreciates due to weather conditions but mainly by heavy truck traffic. Capital stock depreciation rate is assumed to be 0.02. Depreciation rate for road infrastructure,  $\delta_g$ , is calculated according to OECD methodology in OECD (2001) with a value set equal to 0.015 (this implies a 4,43% depreciation rate annually). Boskin *et al.* (1989) assume that roads depreciate geometrically at a rate of 1.98 percent per year.

*Travel and car use* ( $\eta_1, \delta_q$ ): Following Parry *et al.* (2014), we set a value for  $\eta_1$  equal to 4, which according to Small and Verhoef (2007) is a value consistent with the Bureau of Public Roads formula, used traditionally to predict vehicle speed. Empirical studies suggest that this value is in the range 2.5 to 5, depending on the size of the urban centre considered. The larger the urban centre, the higher the value.<sup>12</sup> Depreciation rate for cars is calculated by assuming an average lifespan of 12 years, which result in a quarterly depreciation rate,  $\delta_q$  equal to 0.0208 (0.083 in annual terms).

$$\delta_q = \frac{1}{4 \times 12} = 0.0208. \quad (42)$$

Wei (2013) set the maximum life span of vehicles to be equal to 15 years and then the depreciation rate is set at an annual rate of 0.1.

*Exogenous prices* ( $p^F, p^{MR}$ ): We consider prices related to fuel consumption and maintenance and repairs of cars as exogenous given. The other two prices are calibrated endogenously. We normalize the price of maintenance and repairs,  $p^{MR}$  to 1. The price of fuel,  $p^F$ , is fixed to 3.15, following data from the Energy Information Agency (EIA).

*Tax rates* ( $\tau^F, \tau^x$ ): The fuel tax rate,  $\tau^F$ , is set equal to 0.495, following data from the Energy Information Agency (EIA). The tax on cars' purchase price is fixed to be 20%.

*Exogenous shocks* ( $\zeta_A, \sigma_A, \zeta_F, \sigma_F, \zeta_X, \sigma_X$ ): Parameters driving the AR(1) processes defined for TFP, oil price shocks, and cars sales shocks are estimated by OLS. For the oil shock, we use data from table 2.3.4. from BEA. We take line 11: Gasoline and other energy goods, as the reference series to estimate oil price shocks. For cars sales shocks we use data from Consumer Price Index: New Vehicles, from BLS. Data has a quarterly frequency and the sample period is 1970:Q1 to 2014:Q4. Estimated values are 0.952 for the persistence parameters of the AR(1) process,  $\zeta_F$ , and 0.075 for the standard deviation,  $\sigma_F$ .

### 3.3. Parameters calibrated internally

Next, we calibrate the parameters that require solving the model. We summarize them in Table 3. The preference parameter representing the willingness to work,  $\varphi_h$ , is chosen to match average household hours worked in the market  $H^* = \frac{8}{24} = \frac{1}{3}$ , given the steady state value of consumption,  $C^*/Y^* = 0.7362$ , and driving time.

<sup>12</sup>See Vickrey (1963), Small and Verhoef (2007) and Small (1992a,b).



Parry *et al.* (2014) estimate a surface road capacity of 17.3 km. per vehicle. Given that the value for  $Q^*$  is 1.7822, this implies that the stock of roads is 30.8321 kilometers per household. Measuring the stock of roads in term of output, given the value for road investment and road depreciation rate, in steady state the stock of road is calculated to be:

$$K_g^* = \frac{I_g^*}{\delta_g} = \frac{0.0148}{0.015} = 0.9867. \quad (43)$$

Similarly, the steady state stock of capital is given by:

$$K^* = \frac{I^*}{\delta_k} = \frac{0.1766}{0.015} = 11.7730. \quad (44)$$

Following Bento *et al.* (2009), for all households and cars, the average elasticity,  $\epsilon_{oc}^{km}$ , is set equal to 0.74. It is lower for new cars than for older vehicles. Therefore, the parameters measuring the decreasing marginal returns from vehicle services,  $\phi$ , and the parameter measuring the constant relative risk aversion for car services,  $\gamma$ , are related such as:

$$\epsilon_{oc}^{km} = \frac{1}{1 - \phi(1 - \gamma)} = 0.74 \quad (45)$$

Estimates of relative risk aversion are commonly set between one and two. From the steady state equilibrium conditions for driving decision and car investment decision, derived from equations (27) and (28) we obtain that the following expression for the price of cars:

$$(1 + \tau^x) p^x = \frac{\beta(1 - \phi) oM - \phi p^{TI}}{\phi(1 - \beta(1 - \delta_q))} \quad (46)$$

Solving for  $\phi$  in the above expression, we obtain a value of 0.4077. Therefore,  $\gamma$  is set equal to 1.8618 and  $\varphi_s$  is set equal to 0.2542 which also matches the fraction of new cars investment over GNP, equal to 0.04 according to data from the Bureau of Economic Analysis:

$$\frac{p^x X^*}{Y^*} = 0.0291 \quad (47)$$

Given the calibrated value for cars depreciation rate ( $\delta_q = 0.0208$ ), from steady state equation we obtain that:

$$X^* = Q^* \delta_q = 0.0371, \quad (48)$$

and given  $Q^*$ , this results in:

$$p^x = \frac{0.0291}{0.0371} = 0.7837 \quad (49)$$

The remaining price, the price of tolls and insurance, is also calibrated internally using the following expression:

$$p^{TI} = \frac{TI^*}{Q^*} = 0.0039 \quad (50)$$

According to these assumptions, the steady state condition derived from equation (26) gives:

$$\varphi_h = (1 - \alpha) \theta \frac{Y^*}{C^* H^* (H^* + D^*)^{1/\nu}} = 9.7659. \quad (51)$$

From the steady state condition for VKT of the model we also obtain that:

$$\varphi_s = \frac{o_t - \phi(1 - \theta)W_t H^\theta M^{\phi(1-\theta)-1} Q^{-\theta}}{\phi M^{\phi(1-\gamma)-1} Q^{-\gamma} C} = 0.2542. \quad (52)$$

Given our definition of congestion, driving time is defined using the standard power function:

$$D^* = \eta_0 \left( \frac{M^* Q^*}{K_g^*} \right)^{\eta_1} = 0.0452 \quad (53)$$

From that expression, we compute the value for the scale parameter  $\eta_0$ , given that  $M^* Q^* = 32.97$ ,  $K_g^* = 0.5333$ , and  $\eta_1 = 2.5$ , we obtain a value of  $7 \times 10^{-4}$ . Notice that our measure of congestion in steady state is just  $32.97/0.9867 = 33.42$ .

Consumption of gasoline liters per kilometer is calculated as:

$$\omega_F = \frac{F^*}{((p^F + \tau^F)M^* Q^*)} = 1.67 \times 10^{-4}. \quad (54)$$

Similarly, the proportionality parameters of vehicle maintenance and repairs to kilometers driven is calculated as:

$$\omega_{MR} = \frac{Z^*}{(p^{MR}M^* Q^*)} = 4.91 \times 10^{-4}. \quad (55)$$

Given these parameters, the operating cost per kilometer driven is calculated to be  $o = 0.0011$ .

Capital income over total income in steady state is given by:

$$\frac{R^* K^*}{Y^*} = \alpha \frac{\mu (K^*)^\rho}{\mu (K^*)^\rho + (1 - \mu) (K_g^*)^\rho}, \quad (56)$$

From that expression we calculate the value for  $\mu = 0.9463$ .

Finally, Total Factor Productivity is obtained as a residual from the aggregate production function as:

$$A^* = \frac{Y^*}{\left[ \mu (K^*)^\rho + (1 - \mu) (K_g^*)^\rho \right]^{\alpha/\rho} (\bar{H}_t^*)^{1-\alpha}} = 0.8517. \quad (57)$$

**Table 3: Internally calibrated parameters**

Category	Notation	Parameter definition	Value
Preferences	$\psi_h$	Willingness to work	9.7659
	$\psi_s$	Willingness to drive	0.2542
Technology	$\mu$	CES weight for private capital	0.9463
	$A$	Total Factor Productivity	0.8517
Travel and car use	$\omega^F$	Gasoline liters per kilometer	$1.67 \times 10^{-4}$
	$\omega^{MR}$	Cars maintenance and repairs	$4.91 \times 10^{-4}$
	$\eta_0$	Congestion scaling parameter	$7 \times 10^{-4}$
	$\phi$	Diminishing returns of car use	0.4676
	$\gamma$	CRRA for car use	1.8618
	$o$	Operating cost per km. driven	0.0011
Prices	$p^x$	Price of cars	0.7837
	$p^{TI}$	Price of tolls and insurance	0.0039
Road investment	$\xi$	Fraction of fiscal income to road	1.0000

#### 4. Model Economy Dynamics

In this section we present some simulations to study the dynamics of our model economy via impulse-response functions to different shocks. In particular, we are interesting in study three different shocks that can be central in studying the relationship between macroeconomic variables and road traffic related variables. First, we consider a total factor productivity shock (neutral technological shock) to the economy to identify how traffic and driving decisions reacts to the business cycle, and the implications of considering cars as a working tool complementary of raw labor. Second, we study the effects of a shock to the fuel price, as fuel taxes are usually proposed as a Pigouvian instrument to internalizing the social cost of congestion. Finally, we study the effects of a shock to the price of cars, as also the characteristics of a car sale tax will be studied as an instrument to internalizing the costs of congestion.

##### 4.1. Aggregate productivity shock

Our first exercise considers the case of an exogenous positive neutral shock to the economy, that is, an increase in Total Factor Productivity,  $A_t$ . This is a standard shock studied in most real business cycle models and so it is used as a benchmark. The idea is to study how the economy reacts to a productivity shock when traffic, cars investment, driving time, and public investment in road are added to the standard model. First, we conduct the analysis conditioned to the fact that road public investment is a fixed proportion of fiscal revenues. This implies that a positive productivity shock increases fiscal revenues (this is always true, as we only consider value added taxes or lump-sum taxes). As fiscal revenues increases, road investment also increases. Next, we isolate the effects of the productivity shock to the case in which the road stock is fixed and fiscal revenues are not related to road investment. Dynamic responses of the variables are similar in both contexts, and hence, we present the result only for the second case (constant stock of roads).

We assume that TFP increases by one standard deviation on impact. As expected, this shock raises output on impact, as more output is produced for given factor inputs. Private investment

also increases in the period when the shock occurs as this shock reduces the marginal cost of capital accumulation. As a consequence, the private capital stock also increases given the rise in its productivity, increasing the persistence of output to the shock. These changes in output and physical capital lead to a gradual increase in consumption above its steady state. Public investment in road is assumed to increase when a fixed proportion of fiscal revenues is assumed to be expended on road investment. However, results are similar when a constant stock of road is assumed. Thus, from a qualitative point of view, the overall effects of this shock in our theoretical framework are the same than in the standard real business cycle model without traffic and roads: a rise on output, investment, consumption and capital stock in response to the shock. The main difference is observed in the response of labor. The model produces a negative response of working hours to the aggregate productivity shock following an initial positive impact. However, the most important implication is that, given the existing relationships between macroeconomic variables and road traffic variables in our model economy, the quantitative effects of a positive aggregate productivity shock are reduced, indicating that road traffic conditions following this shock poses a limitation to the expansion of economic activity.

The inclusion of roads and traffic in the households utility function and in the aggregate production function allows us to move beyond the standard results and observe the relationship between traffic-related variables and the business cycle. The most important finding is that aggregate productivity shock also impacts positively on VKT, and initially the level of congestion increases, rising the travel time and the generalized costs of traveling. The impact is also positive in the acquisition of new cars. The rise of the VKT is observed in spite that the flow kilometers driven,  $M$ , decreases after an initial positive response, given that the current stock of cars,  $Q$ , increases more than proportionally.

The impact of an increase in TFP on the main macroeconomic variables is summarized in Figure 1. We plot deviations in percentage points from the steady state values. The shock has a positive effect on output, as expected, and described above. Both consumption and private investment increase on impact but through different channels. On the other hand, the shock increases the rental rate of capital, thus increasing private investment. Labor services increase in impact, but after some periods the response turn out to be negative. This is provoked by a negative response of working hours given that the response of vehicle services is also positive.

Figure 2 summarized the impulse-response functions for the variables related to cars use. The number of kilometers traveled increases in impact over its steady state value although after some periods the behavior turn out to be negative. The rise in cars investment transforms in a rise in the stock of cars. Given the response of these two variables, the vehicle-kilometers traveled also increases provoking a rise in congestion. Importantly, it is this rise in road congestion the factor behind the reduction in both working hours and leisure. As a conclusion, the model predicts a rise in road congestion as a consequence of a positive aggregate productivity shock. Therefore, the model indicates that traffic flow is pro-cyclical, moving in the same direction that economic activity. Given symmetry in the effects of shocks, our model economy predicts a reduction in road traffic during downturns. The higher level of traffic is produced initially by a larger value of kilometers driven but afterward it is lead by the rise in the stock of cars. Nevertheless, the higher level of road congestion has a negative impact on the economy, mitigating the positive effects of the productivity shock. In other words, more time is spent in driving, reducing the available time

for both working activities and leisure.

Table 4 shows the correlations between output and the traffic related variables. Correlation between output and VKT is close to 1, indicating that economic activity is the driving force for traffic in the short-run, consistently with the assumption of the model that driving time is related to working activities. Both the stock of cars and cars investment show correlations about 0.7 with output. However, the number of kilometers driven display a low correlation with output (0.28), indicating that most of generated traffic corresponds to a rise in the number of cars. That is, when a productivity shock hits the economy, the main effects are observed on the number of cars and in a lesser extend in the number of kilometers driven. This is expected, as in steady state, the stock of cars is determined to maximize vehicle services, which depends on both flow traffic and the stock of cars. Notice that correlation between kilometers driven and the stock of cars is negative (-0.44), indicating that the overall effects is to increases the number of cars and reduce their use. Interestingly, correlation between driving time and the stock of cars is very low (around 0.1), indicating that congestion is mainly explained by the number of kilometers driven. That is, what provokes congestion in roads is not the stock of cars but their use.

	$Y$	$X$	$Q$	$M$	$VKT$	$D$
$Y$	1					
$X$	0.7353	1				
$Q$	0.7345	0.4325	1			
$M$	0.2856	0.4016	-0.4393	1		
$VKT$	0.9982	0.7297	0.7732	0.2292	1	
$D$	0.7152	0.6774	0.0968	0.8119	0.6766	1

#### 4.2. Fuel price shock

Next, we use our model economy to study the effects of a positive fuel price shock. The literature had studied extensively the welfare implication of Pigouvian taxes as an instrument to mitigate negative externalities arising from congestion. Therefore, we expect that a positive fuel price shock helps to increases social welfare reducing traffic flows and thus congestion due to the higher operative travel costs. However, whereas the effect on social welfare is positive, the effect of this shock on the economic activity are unknown. In principle, we could expect a negative effect of a rise in fuel prices over economic activity as in our theoretical framework driving is a compulsory activity for producing and the rise in fuel price means a rise in driving operative costs. However, under the presence of congestion a rise in fuel prices can help to reduce the costs of congestion, thus having a positive impact on economic activity. Therefore, the final effects on economic activity of a rise in fuel prices could be positive or negative, depending on its capacity in internalizing the costs of congestion. Our model economy is a laboratory in which the final economic implications of this shock can be identified.

Figures 3 and 4 plots the impulse-response function following a positive fuel price shock. As expected, this shock implies a rise in operative costs of driving, reducing the number of the flow

kilometers driven. The impact of this shock on vehicle kilometers driven ( $VKT$ ) depends not only on the response of kilometers driven ( $M$ ), but also in the stock of cars, ( $Q$ ). The final impact is a less intensity use of cars. Indeed, we find that the stock of cars responds positively to this shock. This is because this shock has a positive instantaneous effects on new cars investment. The rise of the stock of cars following a rise in the fuel prices could be surprisingly but it is explained from the optimizing behavior of households. Households obtain utility from driving as vehicles produce valuable services. Nevertheless, this shock reduces the vehicle services as the operative costs of driving increase. Given the reduction in the kilometers driven resulting from the raise in driving operative costs the alternative for mitigating the negative effects of this shock in utility vehicle services is by increasing the stock of cars.

As a consequence, there is a reduction in the level of congestion which implies a saving in driving time. This reduction in the congestion is expected as the literature has proposed the use of fuel taxes to internalizing the costs of congestion. Importantly, the observed reduction in congestion is not due to a reduction in the stock of cars. The remaining key question is how are the final effect on economic activity. On the one hand, the rise in fuel prices provoke a increases in driving operative costs, affecting negatively to economic activity. On the other hand, the reduction of congestion saves time, a fraction of which is used in working activities with a positive impact on economic activity. Furthermore, the rise in the stock of cars also has a positive impact on labor in efficiency units.

Another result derived from our model economy consists in the observed instantaneous substitution between physical capital investment and new cars acquisition. In the simulation we find that the final effect on output is positive. Additionally, it is also a positive response on consumption. The only negative effect is the instantaneous negative impact on investment. However, investment increases after the impact, increasing the stock of physical capital. The negative initial response of investment in physical capital is explained by a similar initial response in new cars acquisition in the opposite direction. The shock provokes an instantaneous redistribution of saving between new cars acquisition and new physical capital acquisition.

In summary, the effects of a shock to fuel price on economic activity depends on the level of congestion. Without congestion, we should expect that a rise in fuel prices would have negative effects on economic activity, as driving operative costs increase. However, in an environment of congestion, the rise in fuel prices can have a final positive effect on economic activity as this shock reduces congestion.

### *4.3. Cars price shock*

Finally, the model can be used to investigate the effects of a positive shock to the prices of cars. In our model economy, cars are not only a durable consumption good producing services, but they are also considered as a work tool complementary to working time. Therefore, price of cars is not a factor affecting only traffic-related variables and households' decisions on investing in this durable consumption good, but also has implications over economic activity given that cars is a component of hours worked in efficiency units. Furthermore, given our specification of the households' budget constraint total saving can be divided between the investment in physical capital and the acquisition of new cars. A related question to be studied in Section 6 is whether car sales taxes can be used as an instrument to internalizing congestion costs.

Figures 5 and 6 plot the impulse-response of the relevant variables following this shock. As expected, this shock has an impact on the decision about acquisition of new cars. We find that this shock provokes an instantaneous adjustment in car investment in the period following the shock (a sudden stop in acquisition of new cars), followed by the returning new cars acquisition to the steady state value, in spite of the persistence of the shock. The direct consequence of this sudden stop in new cars acquisition reduces the total stock of cars. Indeed the stock of cars will be below its steady state value during the persistence of the shock.<sup>13</sup>

Parallel to the reduction in new cars investment and in the stock of cars, it is a rise in their use intensity. We observe a positive response in the flow kilometers driven. Given the rise in the price of cars and the reduction in the stock of cars, households react by increasing their use as a way to maximize utility by mitigating the negative effect of the shock on vehicle services. In fact, this shock has a negative impact on services from vehicles, indicating that the rise in the flow kilometers driven is not enough to compensate the down in the stock of cars. In summary, the response of households with respect to car use intensity has important consequences on traffic. The shock impacts positively on *VKT*, increasing also congestion. This is explained by a higher positive reaction if the kilometers driven compared to the reduction in the stock of cars.

Apart from the effects of this shock on traffic related variables, this shock has also important effects on capital investment decision. The most important effect is the observed substitution of investing in new cars by investment in physical capital. This shock provokes an instantaneous reduction in the acquisition of new cars accompanied by a similar positive movement in physical capital investment. After the initial adjustment in both variables, they return to their steady state values. Finally, the effect of a rise in the prices of cars provokes a reduction in final output and also in labor in efficiency units. As noted above, the instantaneous reduction in the acquisition of new cars implies a reduction in the stock of cars which translate to a down in labor input. Interestingly, the relationship between wages and working hours is negative following this shock. The response of consumption is positive initially but after some periods it turns negative.

## **5. Road investment and generated traffic: The fundamental law of road congestion**

In principle, one might think that congestion problems can be solved by investing in new roads as existing capacity is used. Nevertheless, as have been pointed out by Downs (1962) as road capacity increases, traffic also increases in the same proportion as to congestion remains constant. This is the so-called "Fundamental Law of Road Congestion". This law was initially suggested by Downs (1962, 1992) and basically says that vehicle kilometers traveled increases proportionately to roadway lane kilometers for highways. This implies that it is not possible to reduce congestion by building more roads as more traffic is generated in response to the new road capacity.

There is a vast empirical literature studying the relationship between road capacity expansion and traffic. One important result in this literature is that road capacity expansions cannot reduce congestion because new roads generate additional traffic given the existence of a latent demand.

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<sup>13</sup>In our model economy cars depreciation rate is constant. This means that average aging of the stock of cars is also constant. Variable average aging of the stock of cars can be easily incorporated into the model by considering a non-constant depreciation rate for cars.

This generated traffic reduces or even cancel out the effects of construction new road capacity as an instrument to reduce congestion. Generated traffic in response to road capacity expansion comes from two sources (Litman, 2016): Diverted traffic and induced traffic. Diverted traffic is due to trips shifted in time, route and destination, what Downs, 1992, call the *Triple Convergence*. Induced traffic is related to shifts from other modes, longer trips and new trips. By contrast, Baum-Snow (2007) argues that new highways has contributed to central city population decline, as people re-optimize location choice. As pointed out by Hymel, Small and Van Dender (2010) any policy with the objective of reducing congestion without otherwise making driving more expensive, will have the opposite effect by attracting more traffic. All these results had raised a number of paradox relating road investment with traffic, as the Downs-Thomson paradox, the Braess paradox or the Marchetti's constant, all of them indicating that an improvement (reduction) of the road capacity will attract (reduce) additional traffic flows.

The Fundamental Law of Traffic Congestion has an important economic policy recommendation: It is not possible to relieve road congestion by building more roads. In fact, highway construction is expensive and the value of the elasticity of traffic with respect to road improvements is a central question in the debate about new road construction. Based on that Fundamental Law it results clear that generated traffic reduces benefits of road capacity expansion over congestion. Plane (1995) defines what he called the "black hole theory of highway investment" as investment in highways results in greater ease of travel, increasing both the number of trips and the trip length, calling for additional investment in a repeat process.

Jorgensen (1947) was the first to estimate empirically the response of traffic to road capacity expansions for the case of the construction of the Merritt and Wilbut Cross parkways in the corridor between New York city and New Haven, Connecticut. Nevertheless, estimated generated traffic associated to road improvements are very imprecise and estimated elasticity goes from 0 to 100 percent. In general, the majority of empirical estimations only support partially the "Fundamental Law of Highways Congestion". For instance, Hansen (1995) obtains values in the range from 0 to 0.3 one year after improvement and from 0.2 to 0.8 four years later. Using data from the California Metropolitan Areas, he obtains that at the county level a 1 percent increase in lane-miles induces an immediate 0.2 percent increase in traffic, building to a 0.6 percent increases two years after the improvement. Similar values are obtained at the metropolitan level. However, he estimated that only a small fraction of the rise in *VKT* is due to the increase in lane-miles. Cervero (2003), also using data from the California freeway estimates a long-term elasticity of 0.64. Although estimated elasticity is always positive, they are usually below one.<sup>14</sup> Nevertheless, a recent article by Durantón and Turner (2011) estimated the elasticity of vehicle kilometers traveled with respect to lane kilometers, obtaining a value close to one for a number of alternative specifications. This is an empirical evidence in favor of the "fundamental law of highways congestion" as proposed by Downs (1962). Moreover, they suggest that the fundamental law can be extended to a broad class of major urban roads, resulting in a general "fundamental law of road congestion". Based on these results, Durantón and Turner (2011) conclude that road capacity expansion is not an appropriate policy to reduce traffic congestion and that the correct instrument should be congestion pricing.

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<sup>14</sup>For surveys of the empirical literature, see for instance, Goodwing (1996), Cervero and Hansen (2002), Cervero (2003) and Graham and Glaister (2004).



Using a similar approach, Hsu and Zhang (2014) obtain an estimated elasticity between 1.24 and 1.34 for Japan.

Interestingly, Duranton and Turner (2011) do an accounting exercise in order to quantify the relative contribution of each possible sources for the increase in traffic following a road capacity expansion. They consider four sources: Changes in trucking and commercial driving; changes in household driving decision; changes in population; and diversion of traffic. They obtain that between 19 and 29% of total traffic increase is due to trucks and that migration accounts for between 5% and 21%. For the relative importance of changes in household driving decision, they obtain a contribution between 9 and 39% of total traffic increase. Finally, diversion of traffic accounts for between 0 and 10% of total traffic increase.

In this Section we evaluate what the model says about the "Fundamental Law of Road Congestion". For that, we introduce a shock in the road investment decisions from the government. In the model, road investment is an exogenous decision taken by the government. In the central planner problem (what we call the mayor problem), the stock of road is chosen just to maximize social welfare. The exercise will consist in calculating the response of traffic to the rise in the stock of roads. Our model only include two out of the four sources of traffic increase considered by Duranton and Turner (2011), that is, commercial driving and household driving decisions and therefore, we would expect an increase in traffic lower than the increase in road capacity with the consequent reduction in congestion.

Figure 7 plots the dynamics of the variables following a rise in the proportion of GDP devoted to road construction. We assume that road investment increases by 1%. This rise in road investment implies that in the long-run, the stock of roads (our variable for road capacity) will rise also a 1%. We find that the increases in *VKT* is around 0.172 of the rise in the stock of road. Therefore, the model correctly predicts a rise in vehicle-kilometers traveled as a consequence of the increases in lane kilometers of road. The response that we find is very robust to the parameters values.<sup>15</sup> Following Duranton and Turner (2011) results, the sum of commercial plus household driving decision account between 28 and 68% of the increase in *VKT*. Our estimated response is close to the lower value of that range.

The rise in *VKT* in the long-run is based on the rise in the number of cars but not in the number of kilometers driven. In impact, the kilometers driven rises, but as the stock of cars increases, kilometers driven reduces to the previous steady state level. That is, kilometers driven is not a function of the stock of roads, but road capacity determines the stock of cars in the long-run. Given that the increases of *VKT* generates by the model is less proportional with respect to the stock of roads, the level of congestion also reduces, as the model does not considered additional factors affecting induced traffic such as changes in population or diverted traffic.

Two important results derive from the analysis. First, the rise in *VKT* computed is related to the level of economic activity. In the model, the stock of road is an additional input in the production function. The rise in road investment decision by the government increases output in the long-

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<sup>15</sup>To check the robustness of the model we calculate the *VKT* responses to road change as a function of the main parameter of the model. We change the values of two key parameters, the elasticity of *VKT* with respect to operating costs and the elasticity of driving with respect to road traffic congestion. We vary each parameters one at a time. *VKT* responses to road capacity expansion remains constant.

run in about 2.2%. Given that public road investment represents only about 1.5% of GDP, a 1% increases in road investment implies an additional amount of 0.015% of GDP. Hence, the implicit long-run output multiplier of public spending in highways is of a magnitude of around 14.67. This result is consistent with the findings by Fernald (1999). Second, our model isolates the effects on *VKT* of population growth and changes in location choice and hence, only economic activity-related affects are taken into account. This is a reason why the fundamental law of congestion does not completely holds in our context. The combination of these two elements shows that the relationship between congestion and road capacity cannot be used as an argument for public road investment decisions.

## 6. Optimal traffic density and optimal congestion: The Mayor's problem

The welfare effects of traffic congestion has been thoroughly analyzed in the literature. Drivers do not take into their own effect on congestion when taking driving decisions. As they only cares about their own costs, efficiency requires a Pigouvian tax equal to the gap between the marginal cost to all drivers and the cost for the individual. Seminal paper is those of Walters (1961) and Vickrey (1963, 1969). This literature has focused on the used of optimal tolls as an optimal instrument to solve congestion problem. Examples are Arnott *et al.* (1990, 1993, 1994).

Walters (1961) uses data of traffic flow and velocity for the highways in the U.S., and find that efficient prices are generally much higher than present levels, which implies that gasoline taxes should be increased and special tolls charged in congested areas. He suggests a minimum fuel tax of 33 cents a gallon, supplemented by urban mileage taxes (10-15 cents a mile for peak traffic and 4-8 cents a mile for off-peak daytime traffic), and special tolls. Parry and Small (2005), taking into account all external costs of traffic (congestion, accidents, and air pollution), obtain that the optimal gasoline tax in the U.S. is 1.01 dollars per gallon, and 1.34 dollars per gallon for the U.K. However, as Parry, Walls and Harrington (2007) pointed out, a fuel tax rise driving costs for all regions at all times being a very blunt instrument for alleviating traffic congestion which is very specific to rush hour periods in urban ares. Instead, the ideal instrument is a road-specific congestion toll that varies with time a day and advances in electronic metering technology make it feasible. Parry *et al.* (2014) calculate travel delays costs by estimating the relationship between travel delays and different transportation indicators and using evidence of the relationship between wages and how people value travel time. Under the assumption that the Fundamental Law of Road Congestion holds, the effect of road investment on congestion is very limited or, as pointed out by Duranton and Turner (2011) even null, and the only space to alleviate congestion problems is via congestion pricing policies.

In this section we study the level of Pigouvian taxes as a way for measuring the cost of congestion. In our model, we have two taxes: vehicles sales tax and fuel tax. We study the optimal tax for each case. To evaluate the cost of congestion, we first solve the social planner problem in our economy with cars, roads and traffic. The Lagrangian associated to the household's problem is given by

$$\mathcal{L} = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(C_t) + \psi_s \frac{M_t^{\phi(1-\gamma)} Q_t^{1-\gamma} - 1}{1-\gamma} - \psi_h \frac{(H_t + D_t)^{1+1/\nu}}{1+1/\nu} \right),$$

$$\begin{aligned}
& -\vartheta_{1,t} \left[ C_t + p_t^X X_t + I_t + I_{g,t} + \left( (p_t^F \omega_F + p_t^{MR} \omega_{MR}) M_t + p_t^{TI} \right) Q_t - Y_t \right] \\
& -\vartheta_{2,t} \left[ K_{t+1} - (1 - \delta_k) K_t - I_t \right] \\
& -\vartheta_{3,t} \left[ Q_{t+1} - (1 - \delta_q) Q_t - X_t \right] \\
& -\vartheta_{4,t} \left[ K_{g,t+1} - (1 - \delta_g) K_{g,t} - I_{g,t} \right]
\end{aligned}$$

with respect to  $(C_t, H_t, M_t, X_t, I_t, I_{g,t}, K_{t+1}, Q_{t+1}, K_{g,t+1})$ , where

$$\begin{aligned}
Y_t &= A_t \left[ \mu K_t^\rho + (1 - \mu) K_{g,t}^\rho \right]^{\alpha/\rho} \tilde{H}_t^{1-\alpha}, \\
\tilde{H}_t &\equiv H_t^\theta Q_t^{1-\theta}, \\
D_t &\equiv \eta_0 \left( \frac{M_t Q_t}{K_{g,t}} \right)^{\eta_1}.
\end{aligned}$$

The first order condition for hours worked and investment are given by:

$$\varphi_h (H_t + D_t)^{1/\nu} = \theta (1 - \alpha) \frac{Y_t}{C_t H_t}, \quad (58)$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[ \frac{1}{C_{t+1}} \left[ 1 - \delta_k + \alpha \frac{Y_{t+1}}{K_{t+1}} \frac{\mu K_{t+1}^\rho}{\mu K_{t+1}^\rho + (1 - \mu) K_{g,t+1}^\rho} \right] \right]. \quad (59)$$

These two expressions do not differ from their competitive counterparts.

A key distinction in the social planner assignment arise in the first order conditions for kilometers driven and vehicles stock, which now internalize the cost driving time due to congestion. From expression, kilometers traveled is given by:

$$\varphi_s \phi M_t^{\phi(1-\gamma)-1} Q_t^{1-\gamma} - \varphi_h (H_t + D_t)^{1/\nu} \eta_0 \eta_1 \frac{Q_t}{K_{g,t}} \left( \frac{M_t Q_t}{K_{g,t}} \right)^{\eta_1-1} \quad (60)$$

$$= \left( p_t^F \omega_F + p_t^{MR} \omega_{MR} \right) \frac{Q_t}{C_t} - \phi (1 - \alpha) (1 - \theta) \frac{Y_t}{C_t M_t}. \quad (61)$$

The equilibrium condition corresponding to vehicle investment is given by

$$\frac{p_t^X}{C_t} = \beta \mathbb{E}_t \left[ \frac{p_{t+1}^X}{C_{t+1}} (1 - \delta_q) + \varphi_s M_{t+1}^{\phi(1-\gamma)} Q_{t+1}^{-\gamma} \right] \quad (62)$$

$$- \varphi_h (H_{t+1} + D_{t+1})^{1/\nu} \eta_0 \eta_1 \frac{M_{t+1}}{K_{g,t+1}} \left( \frac{M_{t+1} Q_{t+1}}{K_{g,t+1}} \right)^{\eta_1-1} \quad (63)$$

$$- \frac{1}{C_{t+1}} \left[ \left( p_{t+1}^F \omega_F + p_{t+1}^{MR} \omega_{MR} \right) M_{t+1} + p_{t+1}^{TI} - (1 - \theta) (1 - \alpha) \frac{Y_{t+1}}{Q_{t+1}} \right]. \quad (64)$$

Finally, the first order condition which determines the optimal amount of road stock is given by the following dynamic Euler expression:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left\{ \varphi_h (H_{t+1} + D_{t+1})^{1/\nu} \eta_1 \frac{D_{t+1}}{K_{g,t+1}} + \frac{1}{C_{t+1}} \left[ 1 - \delta_g + \alpha \frac{Y_{t+1}}{K_{g,t+1}} \frac{(1 - \mu) K_{g,t+1}^\rho}{\mu K_{t+1}^\rho + (1 - \mu) K_{g,t+1}^\rho} \right] \right\}. \quad (65)$$

### 6.1. Pigouvian taxation

Consider the conditions that determine the driving decisions under a competitive environment and under a centralized framework, expressions (27) and (61), respectively. Subtracting both conditions, we reach the following expression for the Pigouvian fuel taxation:

$$\tau_t^F = \frac{\varphi_h (H_t + D_t)^{1/v}}{1/C_t} \eta_1 \frac{D_t}{\omega_F Q_t M_t}. \quad (66)$$

Hence, the Pigouvian tax internalizes the external cost of road congestion, given in the right hand side of (66). The higher the willingness to work ( $\varphi_h$ ) and the fuel efficiency (i.e. the lower  $\omega_F$ ), the higher the taxation required on fuel. Interestingly, the optimal tax depends on the parameter  $\eta_1$ .

Assume next that the fuel tax is fixed according to (66). Then, *under this assumption*, comparing the expressions that determine the new cars purchases under a competitive environment (28), and under a centralized framework (65), the Pigouvian sale tax on new cars purchases should be set according to the following rule:

$$\tau_t^X \frac{p_t^X}{C_t} = \beta (1 - \delta_q) \mathbb{E}_t \left[ \tau_{t+1}^X \frac{p_{t+1}^X}{C_{t+1}} \right]. \quad (67)$$

This implies a Pigouvian sale tax associated to congestion equals to zero at any time  $t$ :

$$\tau_t^X = 0. \quad (68)$$

Note that this implies that the sale tax  $\tau_t^X$  does not help internalize the external cost of congestion. The economic intuition behind this result is simple: congestion is related to the number of drivers using a road at a given moment of time. Therefore, it is related to the use of cars and not to the number of brand new cars and then, a sale tax is not an adequate tool for internalizing congestion costs.

Walters (1961) estimated an optimal fuel tax of a minimum of 33 cents per gallon (8.7 cents per liter). This value is large compared with the state and federal taxes of about nine or ten cents per gallon. Our calibrate fuel ad valorem tax is 0.495, that is, about five times the fuel tax in the Walters (1961) study. Parry and Small (2005) argue that congestion costs range between 1.5 cents and 9.0 cents per mile, with 3.5 cents being the assumed central marginal cost of congestion. Harrington (2006) propose a value of 6.5 cents per mile. On the other hand, *Monthly Energy Review* (November 2016, Table 1.8 Motor Vehicle Mileage, Fuel Consumption, and Fuel Economy), compiles an average miles per gallon of 17.40. This referenced figure includes all type of vehicles (passengers vehicles, vans, pickup trucks, sport utility vehicles and heavy-duty trucks). Combining these estimations, it results that the interval for the fuel tax that internalizes the external cost of congestion is in the range of 0.261 and 1.566 dollars per gallon, with a central value of 60.9 cents per gallon.

Our model produces a larger optimal fuel tax compared to previous studies. Optimal fuel tax is quantify to be 2.48 dollars per gallon (65 cents per litre). This difference is explained by the impact of congestion on economic activity and on social welfare as calculated by the model. In our theoretical framework, congestion is associated with waste time, reducing both working time and leisure time. The combination of the effects of congestion on output and on welfare are behind the obtained high value for the optimal fuel tax.

## 7. Conclusions

This paper develops a Dynamic Stochastic General Equilibrium model in which personal transport activities related to work are included in the households utility function, road infrastructure in the production function, and road investment in the government decision problem. Households driving decision enters in the households utility function in two ways. First, households receives services from driving cars. Second, driving time is related to working time and labor is measured in efficient units where cars are considered as a working tools which complement labor hours. Moreover, public investment decisions are transformed in public capital (roads) stock. Finally, road infrastructure is considered as an additional input in the aggregate production function. All in all, this model can describe the interaction between, road capacity, traffic, stock of cars, congestion and the main macroeconomic variables.

Three mains analysis have been done. First, we examine the business cycle properties of the model economy and the relationship between traffic and economic variables. We find that traffic and hence congestion are pro-cyclical variables affecting the effects of a productivity shock on the economy. As driving time is considered as waste time (it is a disutility), congestion reduces the positive effect of a positive productivity shock (and the opposite). The main conclusion is that traffic, or more precisely congestion, is a smoothing variable of business cycle fluctuations. In fact, people had observed how much traffic intensity and congestion is reduced in bad times, whereas congestion problems appear in activity expansion times. This property of the model is consistent with the empirical evidence. Two additional shocks are also studies: A fuel price shock affecting operative costs of driving and a car price shock, affecting the new cars acquisition price. Results indicate a positive effect of the fuel price shock on total output as the reduction in traffic flow dampens the negative effects of congestion on economic activity. Finally, a rise in cars price reduces the acquisition of new cars and affect negatively to output, increasing the kilometers driven and the level of congestion.

Second, the model can be used to study the validity of the so-called "Fundamental Law of Road Congestion". This law, initially formulated by Downs (1962), conjectures that an increase in road capacity produces a traffic rise of same proportion. Several factors contribute to the rise in traffic when road capacity is improved. The model only includes those factors related to the economic activity. The model predicts a long-run rise in VKT associated to road improvements, with an estimated elasticity of around 50 percent. This figure is consistent with the existing empirical evidence which, in general, only support partially the fundamental law of road congestion. That is, our model predicts that congestion cannot be solved (only partially) with investment in additional road capacity but that road investment is a key factor affecting economic activity. Implicit long-run output multiplier of public investment in roads is about 14 percent. Based on these results two important conclusions can be drawn. First, the validity or not of the fundamental law of road congestion is not a relevant issue for road investment decisions. Congestion is not the immediate target for road capacity expansions but travel needs and road investment plannification should not be done with the objective of reducing congestion but to increases accesitivity to destinations. The Fundamental Law of Road Congestion and its related paradoxes clearly indicates that road capacity need to be extended continuously to address the problem of congestion. Second, it seems that road investment can be an important factor driving productivity growth, as previously shown

by Fernald (1999).

Finally, we use the model to obtain an expression for the optimal fuel tax as a way to quantify the costs of congestion. The model indicates a larger cost of congestion compared to previous studies. The optimal fuel tax is 2.48 dollars per gallon (65 cents per litre). These figures are apparently very high, indicating that congestion costs are of great importance when we take into account the interactions between traffic and economic activity. This result is consistent with the findings of the response of the model economy to a fuel price shock.

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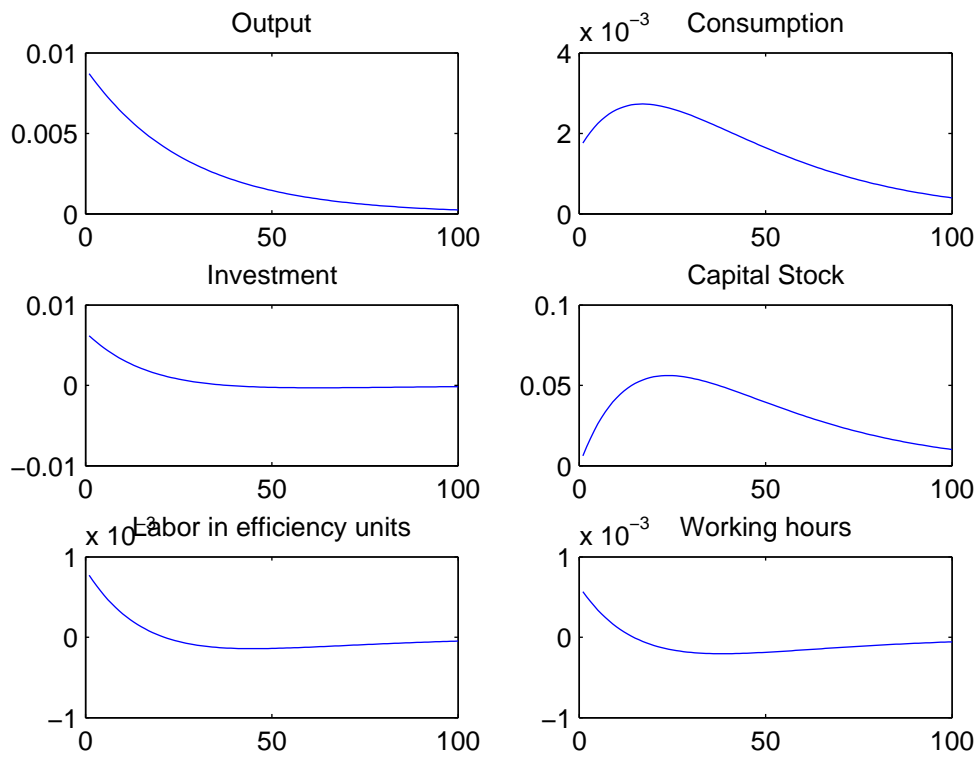


Figure 1: Impulse-response functions: Aggregate productivity shock (I)

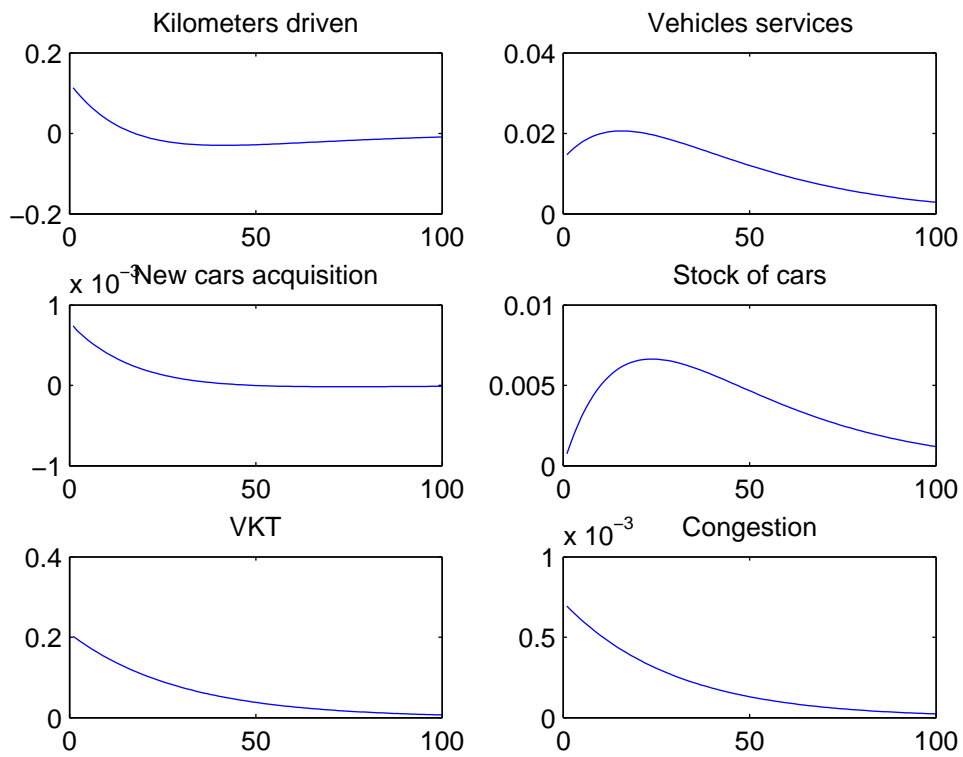


Figure 2: Impulse-response functions: Aggregate productivity shock (II)

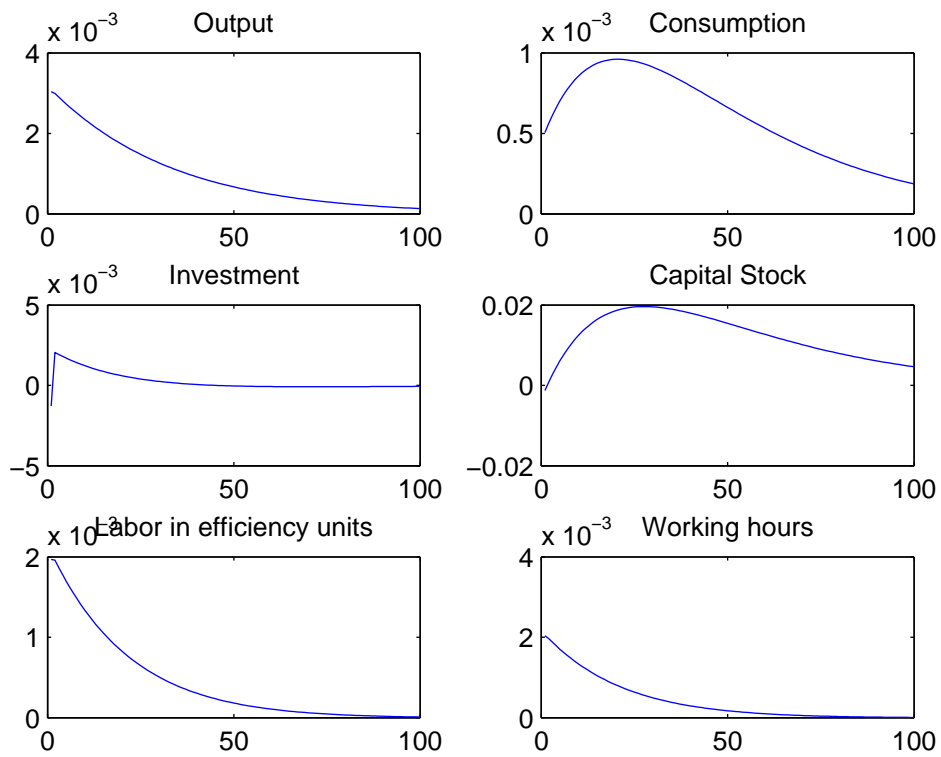


Figure 3: Impulse-response functions: Fuel price shock (I)

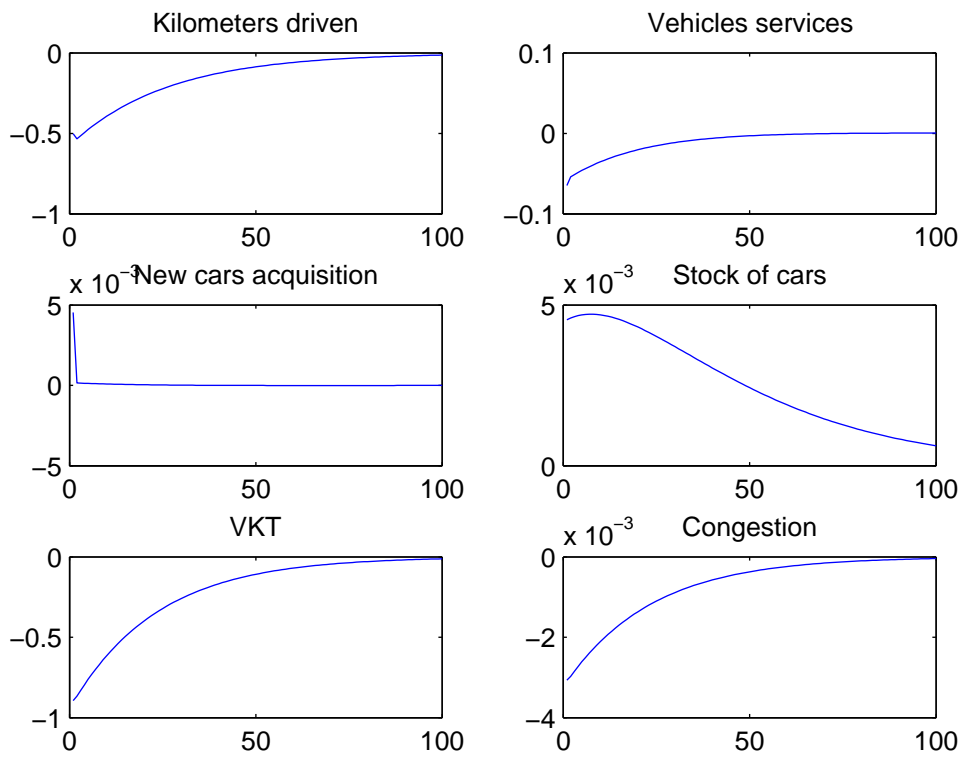


Figure 4: Impulse-response functions: Fuel price shock (II)

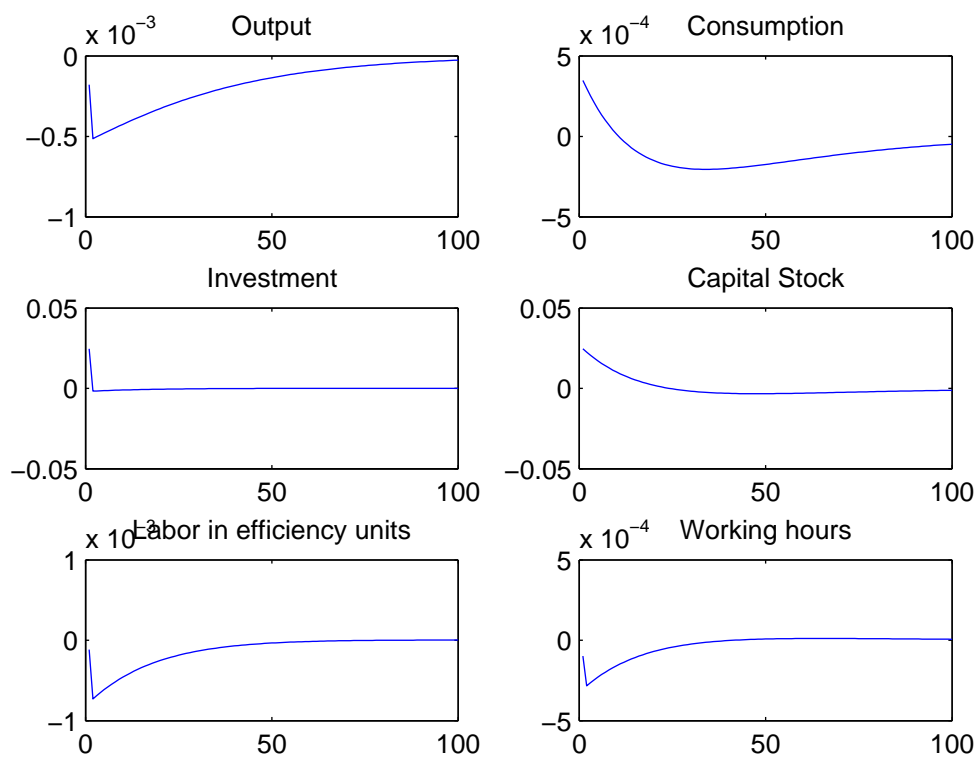


Figure 5: Impulse-response functions: Cars price shock (I)

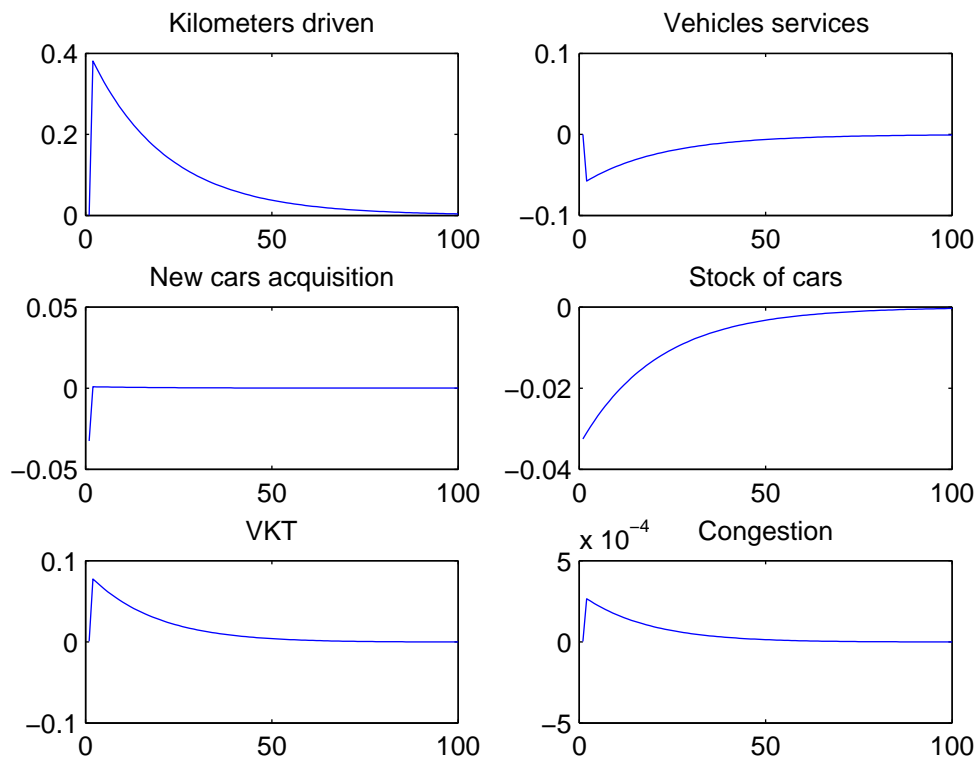


Figure 6: Impulse-response functions: Cars price shock (II)

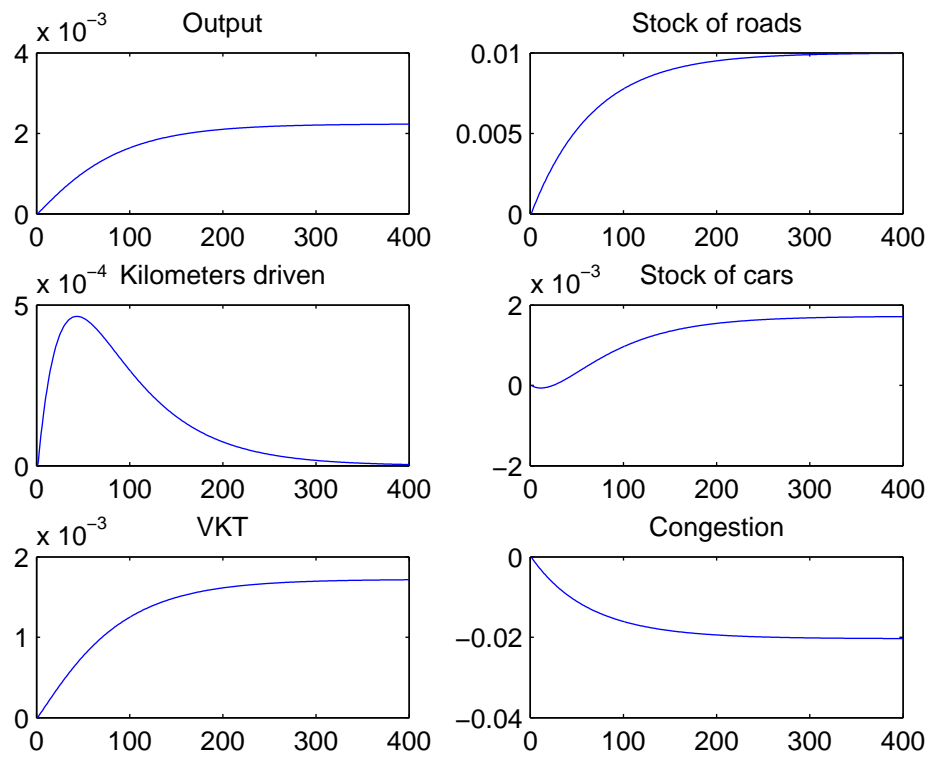


Figure 7: Expansion in the stock of road

# Appendices

## Appendix A. First order conditions that characterize decentralized equilibrium

In a decentralized economy the Lagrangian associated to the household's problem is given by

$$\mathcal{L} = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(c_t) + \varphi_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(h_t + d_t)^{1+1/\nu}}{1+1/\nu} \right), \quad (\text{A.1})$$

$$-\lambda_{1,t} \left[ c_t + (1 + \tau_t^x) p_t^x x_t + i_t + (o_t m_t + p_t^{TI}) q_t - W_t \tilde{h}_t - R_t k_t - \pi_t - TR_t \right] \quad (\text{A.2})$$

$$-\lambda_{2,t} [k_{t+1} - (1 - \delta_k) k_t - i_t] \quad (\text{A.3})$$

$$-\lambda_{3,t} [q_{t+1} - (1 - \delta_q) q_t - x_t], \quad (\text{A.4})$$

with respect to  $(c_t, h_t, m_t, x_t, i_t, k_{t+1}, q_{t+1})$ , where

$$s_t = m_t^\phi q_t, \quad (\text{A.5})$$

$$d_t = \eta_0 \left( \frac{M_t Q_t}{K_{g,t}} \right)^\eta, \quad (\text{A.6})$$

$$\tilde{h}_t = h_t^\theta s_t^{1-\theta} = h_t^\theta m_t^{\phi(1-\theta)} q_t^{1-\theta}, \quad (\text{A.7})$$

$$o_t = (p_t^F + \tau_t^F) \omega_F + p_t^{MR} \omega_{MR}. \quad (\text{A.8})$$

The first order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} : \frac{1}{c_t} - \lambda_{1,t} = 0, \quad (\text{A.9})$$

$$\frac{\partial \mathcal{L}}{\partial h_t} : -\psi_h (h_t + d_t)^{1/\nu} + \lambda_{1,t} \theta \frac{s_t^{1-\theta}}{h_t^{1-\theta}} W_t = 0, \quad (\text{A.10})$$

$$\frac{\partial \mathcal{L}}{\partial x_t} : -\lambda_{1,t} (1 + \tau_t^x) p_t^x + \lambda_{3,t} = 0, \quad (\text{A.11})$$

$$\frac{\partial \mathcal{L}}{\partial m_t} : \varphi_s \phi m_t^{\phi(1-\gamma)-1} q_t^{1-\gamma} - \lambda_{1,t} [o_t q_t - \phi(1-\theta) W_t h_t^\theta m_t^{\phi(1-\theta)-1} q_t^{1-\theta}] = 0, \quad (\text{A.12})$$

$$\frac{\partial \mathcal{L}}{\partial i_t} : -\lambda_{1,t} + \lambda_{2,t} = 0, \quad (\text{A.13})$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : -\lambda_{2,t} + \beta \mathbb{E}_t [\lambda_{1,t+1} R_{t+1} + \lambda_{2,t+1} (1 - \delta_k)] = 0, \quad (\text{A.14})$$

and

$$\frac{\partial \mathcal{L}}{\partial q_{t+1}} : -\lambda_{3,t} + \beta \mathbb{E}_t \left[ \lambda_{3,t+1} (1 - \delta_q) + \varphi_s m_{t+1}^{\phi(1-\gamma)} q_{t+1}^{-\gamma} \right] \quad (\text{A.15})$$

$$-\lambda_{1,t+1} \left[ o_{t+1} m_{t+1} + p_{t+1}^{TI} - (1 - \theta) W_{t+1} \frac{h_{t+1}^\theta}{q_{t+1}^\theta} m_{t+1}^{\phi(1-\theta)} \right] = 0. \quad (\text{A.16})$$



The maximization problem associated with the firm is given by:

$$\max \left\{ A_t \left[ \mu K_t^\rho + (1 - \mu) K_{g,t}^\rho \right]^{\alpha/\rho} \bar{H}_t^{1-\alpha} - W_t \bar{H}_t - R_t K_t \right\}, \quad (\text{A.17})$$

with respect to  $(\bar{H}, K)$ . The first order conditions are given by expressions:

$$\frac{\partial \Pi}{\partial \bar{H}_t} : W_t = (1 - \alpha) \frac{Y_t}{\bar{H}_t}. \quad (\text{A.18})$$

$$\frac{\partial \Pi}{\partial K_t} : R_t = \alpha \frac{Y_t}{K_t} \frac{\mu K_t^\rho}{\mu K_t^\rho + (1 - \mu) K_{g,t}^\rho}. \quad (\text{A.19})$$

where the demand for inputs takes the usual conditions in a competitive economy: the firm will hire labor and capital such that their marginal productivity equates their rental prices,  $W_t$  and  $R_t$ . Profits (unpaid rents to public input) is given by:

$$\pi_t = \alpha Y_t \frac{(1 - \mu) K_{g,t}^\rho}{\mu K_t^\rho + (1 - \mu) K_{g,t}^\rho}. \quad (\text{A.20})$$

## Appendix B. First order conditions that characterize mayor's problem

The Lagrangian associated to the household's problem is given by

$$\mathcal{L} = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(C_t) + \varphi_s \frac{S_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(H_t + D_t)^{1+1/\nu}}{1+1/\nu} \right), \quad (\text{B.1})$$

$$-\vartheta_{1,t} \left[ C_t + p_t^X X_t + I_t + I_{g,t} + \left( (p_t^F \omega_F + p_t^{MR} \omega_{MR}) M_t + p_t^{TI} \right) Q_t - Y_t \right] \quad (\text{B.2})$$

$$-\vartheta_{2,t} \left[ K_{t+1} - (1 - \delta_k) K_t - I_t \right] \quad (\text{B.3})$$

$$-\vartheta_{3,t} \left[ Q_{t+1} - (1 - \delta_q) Q_t - X_t \right] \quad (\text{B.4})$$

$$-\vartheta_{4,t} \left[ K_{g,t+1} - (1 - \delta_g) K_{g,t} - I_{g,t} \right] \quad (\text{B.5})$$

with respect to  $(C_t, H_t, M_t, X_t, I_t, I_{g,t}, K_{t+1}, Q_{t+1}, K_{g,t+1})$ , where

$$Y_t = A_t \left[ \mu K_t^\rho + (1 - \mu) K_{g,t}^\rho \right]^{\alpha/\rho} \bar{H}_t^{1-\alpha}, \quad (\text{B.6})$$

$$\bar{H}_t \equiv H_t^\theta S_t^{1-\theta}, \quad (\text{B.7})$$

$$S_t = M_t^\phi Q_t, \quad (\text{B.8})$$

$$D_t \equiv \eta_0 \left( \frac{M_t Q_t}{K_{g,t}} \right)^\eta. \quad (\text{B.9})$$

The first order conditions for consumption and hours worked are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \frac{1}{C_t} - \vartheta_{1,t} = 0, \quad (\text{B.10})$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : -\varphi_h (H_t + D_t)^{1/\nu} + \vartheta_{1,t} (1 - \alpha) \theta H_t^{\theta(1-\alpha)-1} S_t^{(1-\theta)(1-\alpha)} \quad (\text{B.11})$$

$$+ A_t \left[ \mu K_t^\rho + (1 - \mu) K_{g,t}^\rho \right]^{\alpha/\rho} = 0 \quad (\text{B.12})$$

The first order conditions for new cars investment, miles driven and vehicle stock are given by:

$$\frac{\partial \mathcal{L}}{\partial X_t} : -\vartheta_{1,t} p_t^X + \vartheta_{3,t} = 0, \quad (\text{B.13})$$

$$\frac{\partial \mathcal{L}}{\partial M_t} : \varphi_s \phi M_t^{\phi(1-\gamma)-1} Q_t^{1-\gamma} - \varphi_h (H_t + D_t)^{1/\nu} \eta_0 \eta_1 \frac{Q_t}{K_{g,t}} \left( \frac{M_t Q_t}{K_{g,t}} \right)^{\eta_1-1} \quad (\text{B.14})$$

$$-\vartheta_{1,t} \left[ \left( p_t^F \omega_F + p_t^{MR} \omega_{MR} \right) Q_t - \phi(1-\alpha)(1-\theta) \frac{Y_t}{M_t} \right] = 0, \quad (\text{B.15})$$

$$\frac{\partial \mathcal{L}}{\partial Q_{t+1}} : -\vartheta_{3,t} + \beta \mathbb{E}_t \left[ \vartheta_{3,t+1} (1 - \delta_q) + \varphi_s M_{t+1}^{\phi(1-\gamma)} Q_{t+1}^{-\gamma} \right] \quad (\text{B.16})$$

$$-\varphi_h (H_{t+1} + D_{t+1})^{1/\nu} \eta_1 \frac{D_{t+1}}{Q_{t+1}} \quad (\text{B.17})$$

$$-\vartheta_{1,t+1} \left[ \left[ \left( p_{t+1}^F \omega_F + p_{t+1}^{MR} \omega_{MR} \right) M_{t+1} + p_{t+1}^{TI} \right] - (1-\theta)(1-\alpha) \frac{Y_{t+1}}{Q_{t+1}} \right] = 0. \quad (\text{B.18})$$

The first order conditions for investment and capital stock are given by:

$$\frac{\partial \mathcal{L}}{\partial I_t} : -\vartheta_{1,t} + \vartheta_{2,t} = 0, \quad (\text{B.19})$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : -\vartheta_{2,t} + \beta \mathbb{E}_t \left[ \vartheta_{1,t+1} \frac{\alpha \mu K_{t+1}^\rho}{\mu K_{t+1}^\rho + (1-\mu) K_{g,t+1}^\rho} \frac{Y_{t+1}}{K_{t+1}} + \vartheta_{2,t+1} (1 - \delta_k) \right] = 0. \quad (\text{B.20})$$

The first order conditions for highway investment and road stock are given by:

$$\frac{\partial \mathcal{L}}{\partial I_{g,t}} : -\vartheta_{1,t} + \vartheta_{4,t} = 0, \quad (\text{B.21})$$

$$\frac{\partial \mathcal{L}}{\partial K_{g,t+1}} : \vartheta_{4,t} - \beta \mathbb{E}_t \left[ \varphi_h (H_{t+1} + D_{t+1})^{1/\nu} \eta_1 \frac{D_{t+1}}{K_{g,t+1}} \right] \quad (\text{B.22})$$

$$+ \vartheta_{1,t+1} \left[ \frac{\alpha (1-\mu) K_{g,t+1}^\rho}{\mu K_{t+1}^\rho + (1-\mu) K_{g,t+1}^\rho} \frac{Y_{t+1}}{K_{g,t+1}} + \vartheta_{4,t+1} (1 - \delta_g) \right] = 0. \quad (\text{B.23})$$