The (Minor) Role of Data Revisions on the Estimation of DSGE models?*

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(Preliminary draft)

Abstract

This paper assesses the importance of real time data by estimating an augmented version of the medium-scale New Keynesian model suggested by Smets and Wouters (2007) that considers both revised and real-time data. The estimation results show that ignoring the presence of data revisions of GDP and inflation has not major effects on parameter estimates and model dynamics other than those related to the relative importance of wage push and interest rate innovations in determining the variability of some macroeconomic variables. JEL codes: C32, E30.

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1 Introduction

Aruoba (2008) provides robust evidence that the initial announcements of U.S. aggregate variables are not rational forecast of revised data. In particular, final revisions of output growth and inflation are serially correlated and they are also correlated with real time data initially released by statistical agencies.\(^1\)

This paper studies the importance of considering real time data, beside revised data, in the analysis of DSGE models. Should revisions of real-time data be rational forecast errors, then the arrival of revised data would not be relevant for private agents (households and firms) and policy makers’ decisions, and then parameter estimates would be rather similar using revised, real-time data or both together. The fact that revisions are not rational forecast errors suggests that the analysis of DSGE models based only on revised data could be misleading for two main reasons. From a theoretical perspective, model dynamics could be different when agents take into account that the initial announcements are not rational forecast of revised data. From an empirical perspective, parameter estimates could be biased.

This paper assesses the importance of real time data by estimating an augmented version of the medium-scale New Keynesian (NK) model suggested by Smets and Wouters (2007) that considers both revised and real-time data.\(^2\) The choice of Smets and Wouters (2007) model is well motivated

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\(^1\)There is a long standing literature analyzing the importance of real time data in different contexts. Mankiw, Runkle and Shapiro (1984) is a seminal paper suggesting a theoretical framework for analyzing initial announcements of economic data and applying that framework to the money stock. Diebold and Rudebusch (1991) examine the importance of revision process errors for the index of leading indicators. Orphanides (2001) empirically analyzes the importance of real time data for Taylor rule estimates. Bernanke and Boivin (2003) assess the importance of real time data in a data-rich environment analysis of monetary policy. Croushore and Evans (2006) study the importance of real time data in VAR analyses of monetary policy.

\(^2\)In a similar vein, Vázquez, María-Dolores and Londoño (2010) study the importance of real time data in the context of the basic NK model. An important difference between the two papers is that household and firm choices
because it represents a state-of-the-art model in the monetary economics literature, which builds on Rotemberg and Woodford (1997), and Christiano, Eichenbaum and Evans (2005) models. The augmented version of the NK model assumes that the Fed, households and firms make their decisions taking into account real-time data available, but agents’ decisions determine revised data.

The estimation results show that parameter estimates and impulse response analysis are fairly robust to considering real time data in addition to revised data. However, the variance decomposition analysis is partially affected by considering real time data. These empirical results on the unimportance of real time data are in line with some of the empirical evidence found by Bernanke and Boivin (2003), and Croushore and Evans (2006). The first paper shows that the use of finally revised (as opposed to real-time) data does not seem to matter much regarding the improvement of forecast accuracy when using large data sets. Croushore and Evans (2006) show evidence that the use of revised data in VAR analyses of monetary policy shocks may not be a serious limitation for recursively identified systems. In contrast to Orphanides (2001) who uses a reduced-form approach to estimate Taylor-type policy rules, our empirical results show that the Taylor principle seems to hold when estimating a medium-scale NK model with both revised and real time data.

The rest of the paper is organized as follows. Section 2 describes the extension of Smets and Wouters (2007) model to consider both revised and real time data. Section 3 describes the data are not affected by real-time data issues in the basic model (i.e. decisions by private agents determine the true-revised- values of output and inflation without the need of taking into account real time data), which implies a type of asymmetric information assumption between private agents and the central bank, which uses real time data when implementing monetary policy. This is not the case in the medium-scale New Keynesian model analyzed in this paper. As explained below, there are price and wage indexation rules that force firms (households), which are not able to choose their prices (wages) optimally, to take into account real-time lagged inflation to adjust their prices (wages) instead of revised lagged inflation.

However, they also suggest that the use of real time data may generate some issues when dealing with simultaneous VAR systems.
and the Bayesian estimation procedure. Section 4 discusses the estimation results. Finally, Section 5 concludes.

2 A medium-scale NK model with real-time data

This section builds on the now-standard NK model described in Smets and Wouters (2007), SW henceforth, to accommodate the fact that the Fed, household and firm choices are based on real-time data. More precisely, price and wage indexation rules are based on real time data on inflation available at the time of implementing these indexation rules. Similarly, we consider that the monetary policy rule is implemented with the real-time data available.

The complete loglinearized NK model is presented in the Appendix together with a table describing parameter notation. Here, we focus on explaining how SW model can be modified to take into account real time data on output and inflation. We start by establishing the relationship between the initial announcements of output and inflation and their respective final revised values.

2.1 Revision processes

In the US, the initial announcements of quarterly data on real GDP and the GDP deflator are typically made by statistical agencies with one quarter of delay. Final revisions may take much longer time to be released. Depending upon circumstances, final data on real GDP or inflation may need between 2 and 12 quarters to be released. Subsequently, let us assumed the following generating process for revised output of quarter $t$

$$y_t = y_{t,t+1}^r + rev_{t,t+S}^y,$$  (1)

4 The Bureau of Economic Analysis (BEA) publishes statistical releases of quarterly GDP on a monthly bases. Thus, at the end of January the BEA releases the first estimate of Q4 from last year. By the end of February, the second estimate comes out and, finally, at the end of March (end of Q1), the agency delivers the third estimate.
where \( y_{t,t+1} \) denotes real-time output (released in quarter \( t + 1 \)) and \( \text{rev}_{t,t+S}^{y} \) denotes the revision of output that will be announced in quarter \( t + S \). Similarly, revised inflation, \( \pi_t \), is determined by the following identity

\[
\pi_t \equiv \pi_{t,t+1}^{\pi} + \text{rev}_{t,t+S}^{\pi},
\]  

(2)

where \( \pi_{t,t+1}^{\pi} \) denotes real-time inflation (released in quarter \( t + 1 \)) and \( \text{rev}_{t,t+S}^{\pi} \) denotes the final inflation revision that will be announced in quarter \( t + S \).

Aruoba (2008) finds that data revisions of output growth and inflation are related to the initial announcements of both macroeconomic variables and their revisions. We follow this line of argument to assume that revisions of output and inflation are determined by the following processes

\[
\text{rev}_{t,t+S}^{y} = b_{yy} y_{t,t+1} + b_{y\pi} \pi_{t,t+1}^{\pi} + \varepsilon_{t,t+S}^{y},
\]  

(3)

\[
\text{rev}_{t,t+S}^{\pi} = b_{\pi y} y_{t,t+1} + b_{\pi\pi} \pi_{t,t+1}^{\pi} + \varepsilon_{t,t+S}^{\pi}.
\]  

(4)

These two revision processes are not intended to provide a structural characterization of the revision processes followed by statistical agencies, but to provide a simple framework to assess whether departures from the hypothesis of well-behaved revision processes (i.e. white noise revision processes) might affect the estimates of behavioral and policy parameters. More precisely, these processes allow for the existence of non-zero correlations between output and inflation revisions and the initial announcements of these variables. Moreover, the revision process shocks \( \varepsilon_{t,t+S}^{y} \) and \( \varepsilon_{t,t+S}^{\pi} \) are assumed to follow AR(1) processes, i.e. \( \varepsilon_{t,t+1}^{y} = \rho_{y} \varepsilon_{t-1,t-1+S}^{y} + \eta_{t+S}^{y} \) and \( \varepsilon_{t,t+1}^{\pi} = \rho_{\pi} \varepsilon_{t-1,t-1+S}^{\pi} + \eta_{t+S}^{\pi} \) where both \( \eta_{t+S}^{y} \) and \( \eta_{t+S}^{\pi} \) are white-noise innovations.
2.2 New Keynesian Phillips curve

The separation between real-time data and final data may have an impact on pricing decisions that take into account indexation rules. SW (2007), and may other papers, consider that all the firms that cannot price optimally apply an indexation rule on lagged inflation to adjust their prices. It would lead them to use real-time inflation for price adjustment. Hence, some \( \omega \) firm that would implement the indexation rule would do \( P_t(\omega) = (1 + \pi_{t-1}^r)P_{t-1}(\omega) \) accordingly to the data-extended setup described above, whereas it did \( P_t(\omega) = (1 + \pi_{t-1})P_{t-1}(\omega) \) in Smets and Wouters (2007). If we adopt such real-time price indexation scheme, the loglinearized equation for the optimal price set by firms capable of reoptimizing their prices becomes:

\[
p_t^*(i) = (1 - \beta \xi_p) E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \left( A \left( mc_{t+j}(i) + \lambda^p_{t+j} \right) + p_{t+j} - \pi_{t+k} \sum_{k=1}^{j} \pi_{t+k-1,t+k} \right),
\]

where \( p_t^*(i) \) is the log of the optimal price set by firm \( i \), \( A > 0 \) is a constant parameter that depends upon the Kimball (1995) goods market aggregator and the steady-state price mark-up. The log of the optimal price depends on the expectation of three factors: the log of the real marginal costs, \( mc_{t+j}(i) \), exogenous price mark-up variations, \( \lambda^p_{t+j} \), and the log of the aggregate price level adjusted by the indexation rule, \( p_{t+j} - \pi_{t+k-1,t+k} \) which, in contrast to the SW model, considers that the indexation rule takes into account initial announcements of inflation, \( \pi_{t+k-1,t+k} \), instead of revised inflation, \( \pi_{t+k-1} \). Since \( p_{t+j} = p_t + \sum_{k=1}^{j} \pi_{t+k} \), the following optimal relative price (\( \bar{P}_t^*(i) = p_t^*(i) - p_t \)) obtains:

\[
\bar{P}_t^*(i) = A \left( 1 - \beta \xi_p \right) E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \left( mc_{t+j}(i) + \lambda^p_{t+j} \right) + E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left( \pi_{t+j} - \pi_{t+j-1,t+j} \right).
\]

\(^{5}\text{It should be noticed that real-time data refer to the information set that come with the first three monthly releases mentioned at the end of the Section.}\)


\(^{7}\text{More precise, } A = \left( \left( \phi_p - 1 \right) \varepsilon_p + 1 \right)^{-1} \text{ where } \varepsilon_p \text{ is the curvature of the Kimball aggregator and } \phi_p \text{ is the steady-state price mark-up.}\)
Since all firms choosing the optimal price face the same problem, these firms will set the same optimal price. So the optimal price can be written as

\[
\tilde{P}^*_t - \beta \xi_p E_t \tilde{P}^*_{t+1} = A (1 - \beta \xi_p) (mc_t + \lambda_p^r) + \beta \xi_p E_t (\pi_{t+1} - \pi_{t,t+1}^r) .
\]

(5)

Calvo pricing combined with the indexation rule for prices determine, after loglinearization, that relative optimal prices and the revised and real time inflation rates are related as follows

\[
\tilde{P}^*_t = \frac{\xi_p}{1 - \xi_p} (\pi_t - \pi_{t-1,t}^r) ,
\]

which can be substituted into the left-hand side of equation (5) to obtain after some algebra

\[
\pi_t = \frac{\xi_p}{1 - \xi_p} (\pi_t - \pi_{t-1,t}^r) - \beta \xi_p E_t \pi_{t+1}^r - A \left[ \left( 1 - \beta \frac{\xi_p}{\xi_p} \right) \left( 1 - \xi_p \right) \right] \mu_t^p + (1 + \beta \xi_p) \frac{\xi_p}{1 - \xi_p} \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) .
\]

(6)

The mark-up shock has been re-scale at \( \varepsilon_p^* = A \left( 1 - \xi_p \right) \left( 1 - \xi_p \right) \xi_p \) and -following the SW convention- we have introduced \( \mu_t^p \) as the log deviation of the price mark-up (\( mc_t = -\mu_t^p \)). Notice that when the initial announcement and revised data coincide (i.e. real time data \( \pi_t = \pi_{t,t+1}^r \)) the New Keynesian Phillips curve (6) is identical to the one obtained in SW (their equation (10)) given by

\[
\pi_t = \frac{\xi_p}{1 - \xi_p} \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) .
\]

(7)

Using equations (2)-(3), we obtain after some small algebra that^8

\[
E_t \pi_{t,t+1}^r = B \left[ \pi_t - \frac{\xi_p}{1 + \xi_p} \mu_t^p + \frac{\xi_p}{1 + \xi_p} \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) .
\]

(8)

where \( B = \frac{1 + b_{xy} - b_{xxy} b_{xy}}{(1 + b_{xxy}) (1 + b_{xy})} < 1 \) whenever (i) \( b_{xy} > 0 \), \( b_{xxy} > 0 \), and (ii) \( b_{xy} \) and \( b_{xxy} \) share the same sign. Substituting equation (8) into (6)

\[
\pi_t = \frac{\xi_p}{1 + \beta \xi_p} \pi_{t-1,t}^r + \frac{\beta \xi_p}{1 + \beta \xi_p} E_t \pi_{t+1}^r - A \left[ \left( 1 - \beta \frac{\xi_p}{\xi_p} \right) \left( 1 - \xi_p \right) \right] \mu_t^p + \frac{1 + \beta \xi_p}{1 + \beta \xi_p} \frac{\xi_p}{1 - \xi_p} \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) \left( 1 + \beta \xi_p \right) .
\]

(9)

^8Proof available in a technical appendix.
Comparing equations (7) and (9), we observe that considering data revisions has four type of effects in the NKPC specification. First, lagged inflation, \( \pi_{t-1} \), is replaced by lagged real-time inflation, \( \pi^r_{t-1,t} \). Second, \( y_t \) has a positive influence on current inflation through its impact on inflation revisions. Third, current inflation is also affected by the innovations of data revisions: there is a positive impact from the inflation-revision shock, \( \varepsilon^\pi_t \), and a negative influence of the output revision shock, \( \varepsilon^y_t \). Finally, the slope of the NKPC with data revisions is steeper (i.e. \( A_1 + \beta v^\pi p B(1 - \beta \xi_p)(1 - \xi_p) > A_1 + \beta v^\pi p B(1 - \beta \phi_p)(1 - \xi_p) \)) whenever \( B < 1 \).

2.3 Real wage dynamics

SW (2007) borrow the labor market with wage-setting households and sticky wages of Erceg et al. (2000). It assumes the standard Calvo (1983)-type rigidity for wage adjustments. For non-optimal wage adjustments households follow an indexation rule on lagged inflation, analogous to the one described above for non-optimal price adjustments. In our extension to SW (2007), we are replacing lagged inflation for its real-time observation to write the proportional relationship between relative optimal wages, \( \tilde{W}^*_t \), and the rate of wage inflation adjusted by the indexation factor, \( \pi^w_t - \nu^w \pi^r_{t-1,t} \), as follows

\[
\tilde{W}^*_t = \frac{\xi^w}{1 - \xi^w} (\pi^w_t - \nu^w \pi^r_{t-1,t}),
\]

where \( \xi^w \) is the Calvo probability of not being able to set the optimal wage. In turn, the real wage dynamic equation only departs from the one considered in the SW model in those terms related to the indexation factor (i.e. the terms that include the indexation parameter, \( \nu^w \))

\[
w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi^r_{t+1}) - w_1 \pi_t - w_2 \bar{\nu} w E_t \pi^r_{t,t+1} + w_3 \pi^r_{t-1,t} - w_3 \nu^w \pi^w_t + \varepsilon^w_t, \quad (10)
\]

where

\[
w_1 = \frac{1}{1 + \beta}, \quad w_2 = \frac{\nu^w}{1 + \beta}, \quad \text{and} \quad w_3 = \frac{1}{1 + \beta} \left[ \frac{(1 - \beta)\phi^w_{w-1}}{\xi^w ((\phi^w w - 1) + \varepsilon^w)} \right].
\]

As expected, if \( \pi_t = \pi^r_{t,t+1} \) then equation (10) is identical to equation (13) in SW.
Noticing $E_t \pi_{t,t+1} = B \left[ \pi_t - \frac{b_{\pi} \pi_t}{1 + b_y} y_t - \rho_{\pi} \varepsilon^\pi_t + \frac{b_{\pi} \pi_t}{1 + b_y} \rho_{\pi} \varepsilon^y_t \right]$ in (10) yields (after grouping terms)

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_1 (1 + \beta t_w B) \pi_t + w_2 \pi_{t-1,t} - w_3 \mu^w_t \tag{11}$$

The implications of our data-revision extensions on real wage dynamics are the introduction of real-time lagged inflation instead of lagged inflation ($\pi_{t-1,t}$ replaces $\pi_{t-1}$), the influence of current output through its impact on inflation revisions and also the presence of both data revision innovations, $\varepsilon^\pi_t$ and $\varepsilon^y_t$.

### 2.4 Monetary policy rule

The Taylor-type rule brings lagged inflation and lagged output, but their fully-revised observations will not be released until period $t - 1 + S$. Therefore, we must take their rational expectation to bring along

$$R_t = \rho R_{t-1} + (1 - \rho) [r_{\pi} E_t^{CB} \pi_{t-1} + r_{y} (E_t^{CB} y_{t-1} - y^P_{t-1})] + \varepsilon^R_t,$$

where the superscript "CB" of the conditional expectation operator explicitly tell us that the Central Bank information set at time $t$ is different from the information set of private agents - firms and households- since it only includes the initial announcements of inflation and output, potential output and it takes into account the possibility that revision process are not well-behaved as stated in equation (3) and (4).\footnote{It is assumed that potential output belongs to the information sets of households, firms and the Fed. In order to analyze the importance of this assumption, we have also estimated the extended NK model with real time data by removing potential output from the policy rule. The estimation results are not sensitive to this alternative specification. These results are available from the authors upon request.}

Using the identities (1) and (2) that relate final data to real-time data leads to

$$R_t = \rho R_{t-1} + (1 - \rho) [r_{\pi} E_t^{CB} \pi_{t-1,t} + r_{\pi} E_t^{CB} \pi_{t-1,t-1} + r_{\pi} E_t^{CB} \pi_{t-1,t-1+S} + r_{y} E_t^{CB} y_{t-1,t} + r_{y} E_t^{CB} y_{t-1,t-1+S}] + \varepsilon^R_t,$$
where the generating processes (3) and (4) can be inserted to yield

$$R_t = \rho R_{t-1} + (1-\rho)[r_\pi(E_t^{CB} \pi^r_{t-1,t} + b_{\pi y}E_t^{CB} y^r_{t-1,t} + b_{\pi y}E_t^{CB} \pi^r_{t-1,t} + E_t^{CB} \varepsilon^\pi_{t-1,t-1+S}) + r_y(E_t^{CB} y^r_{t-1,t} + b_{yy}E_t^{CB} y^r_{t-1,t} + b_{yy}E_t^{CB} \pi^r_{t-1,t} + E_t^{CB} \varepsilon^\pi_{t-1,t-1+S} - y^p_{t-1})] + \varepsilon^R_t.$$  

Dropping the rational expectation operators that are not required, recalling the AR(1) series for innovations on data revisions, and grouping terms give

$$R_t = \rho R_{t-1} + (1-\rho)[r_\pi \pi^r_{t-1,t} + r_y y^r_{t-1,t} + r_\pi \rho y S^{-1} \varepsilon^\pi_{t-S,t} + r_y \rho y S^{-1} y_{t-S,t} - r_y y^p_{t-1}] + \varepsilon^R_t.$$  

where $R_\pi = r_\pi(1+b_{\pi \pi}) + r_y b_{\pi y}$ and $R_y = r_\pi b_{\pi y} + r_y(1+b_{yy})$.

As pointed out by Aruoba (2008), the initial announcement of quarterly (monthly) macroeconomic variables corresponding to a particular quarter (month) appears in the vintage of the next quarter (month), roughly 45 (at least 15) days after the end of the quarter (month). Therefore, lagged inflation, $\pi^r_{t-1,t}$, and output, $y^r_{t-1,t}$, directly enter in the policy rule. Moreover, since the initial announcements might not be rational forecasts of revised data, the Fed may take into account this feature to predict the actual revisions of the initial announcements of inflation and output (i.e. $b_{\pi y} y^r_{t-1,t} + b_{\pi \pi} \pi^r_{t-1,t}$ and $b_{yy} y^r_{t-1,t} + b_{\pi y} \pi^r_{t-1,t}$, respectively). Furthermore, revision shocks $-\varepsilon_{t,S,t}^\pi$ and $\varepsilon_{t+1+1,S}^y$ might be persistent and then their expected values $E_{t} \varepsilon_{t+S}^\pi$ and $E_{t} \varepsilon_{t+1+S}^y$, respectively, help to predict the revised values of inflation and output. Taking into account inflation and output identities, equations (2)-(1), it is straightforward to see that under this policy rule specification the Fed is assumed to react to expected revised values of lagged inflation $(\pi^r_{t-1,t} + b_{\pi y} y^r_{t-1,t} + b_{\pi y} \pi^r_{t-1,t} + E_t \varepsilon_{t-1,t-1+S}^\pi)$ and lagged output gap $(y^r_{t-1,t} + b_{yy} y^r_{t-1,t} + b_{yy} \pi^r_{t-1,t} + E_t y^p_{t-1,t-1+S} - y^p_{t-1})$.

The complete model includes nine shock processes. The AR(1) technology shock $\varepsilon^\pi_t = \rho_\varepsilon \varepsilon^\pi_{t-1} + \eta^\pi_t$, the AR(1) risk premium disturbance that shifts the demand for purchases of consumption and
investment goods $\varepsilon^b_t = \rho_b \varepsilon^b_{t-1} + \eta^b_t$, the exogenous spending shock driven by an AR(1) process with an extra term capturing the potential influence of technology innovations on exogenous spending $\varepsilon^g_t = \rho_g \varepsilon^g_{t-1} + \eta^g_t$, the AR(1) investment shock $\varepsilon^i_t = \rho_i \varepsilon^i_{t-1} + \eta^i_t$, the AR(1) monetary policy shock: $\varepsilon^R_t = \rho_R \varepsilon^R_{t-1} + \eta^R_t$, the ARMA(1,1) price mark-up shock: $\varepsilon^p_t = \rho_p \varepsilon^p_{t-1} + \eta^p_t - \mu_p \eta^p_{t-1}$, the ARMA(1,1) wage shock $\varepsilon^w_t = \rho_w \varepsilon^w_{t-1} + \eta^w_t - \mu_w \eta^w_{t-1}$, the AR(1) inflation revision shock $\varepsilon^\pi_{t,t+1} = \rho^\pi \varepsilon_{t-1,t-1}^\pi + \eta^\pi_{t,t+1}$ and the AR(1) output revision shock $\varepsilon^y_{t,t+1} = \rho^y \varepsilon_{t-1,t-1}^y + \eta^y_{t,t+1}$. The latter two shocks are introduced in a SW-type DSGE model to study the business cycle implications of data revisions. Notice that model’s solutions depends on $E_t \varepsilon^\pi_{t,t+1}$ and $E_t \varepsilon^y_{t,t+1}$ and these two values depend on the number of periods, say $S$, after which there are no more revisions for each variable other that benchmark revisions that take place occasionally and involve changing methodologies or statistical changes such as base years. Unfortunately, this number $S$ is not stable neither over time nor across variables. As a compromise, we have solved and estimated the model by assuming that on average the final revisions are obtained after six quarters (i.e. $S = 6$).\textsuperscript{10} The appendix displays the complete set of equations of the model.

3 Data and estimation procedure

We estimate both models with U.S. data from the first quarter of 1983 to the first quarter of 2008. We do not consider more recent revisions to minimize the chance of considering as final revisions some observations that can be still revised in the future. Except for some of the last quarters of the sample, corresponding to the 2007-08 financial crises, this period is characterized by mild

\textsuperscript{10}We have also estimated the model assuming an extreme alternative value for $S = 12$. This value for $S$ is considered by Aruoba (2008) as the maximum number of periods after which there are no more revisions for each variable, except for benchmark revisions. The estimation results are not sensitive to this alternative value of $S$. These estimation results are also available from the authors upon request.
fluctuations (the so-called Great Moderation) of aggregate variables (see Stock and Watson, 2002, among others). Thus, the estimation exercises do not suffer from some potential miss-specification sources, such as parameter instability in both the private sector—for instance, Calvo probabilities (Moreno, 2004)—and the monetary policy reactions to inflation or output. Indeed, some authors argue that a sound monetary policy implementation is the main factor behind the low business cycle volatility in this period (Clarida, Galí and Gertler, 1999).

Regarding the data set, we take as observable variables quarterly time series of the inflation rate, the Federal funds rate, the log of hours worked and the log differences of the real Gross Domestic Product (GDP), real consumption, real investment and the real wage. The rate of inflation is obtained as the first difference of (the log of) the implicit GDP deflator, whereas the real wage is computed as the ratio between nominal compensation per hour and the GDP price deflator. The data were retrieved from the Federal Reserve of St. Louis (FRED2) database. In addition, we consider real-time data on output growth and inflation as reported by the Federal Reserve Bank of Philadelphia. Thus, variables displaying a long-run trend enter the estimation procedure in log differences to extract their stationary business cycle component. In this way, we avoid the well-known measurement error implied by standard filtering treatments. Moreover, considering the growth rates of the initial announcement of GDP and the GDP deflator allow us to isolate our analysis from the presence of benchmark revisions.

The estimation procedure also follows SW. Thus, we consider a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information

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11 See Croushore and Stark (2001) for the details of the real-time data set.
12 More precise, the benchmark revisions for GDP and GDP deflator take place about every five years. Given our 25-year sample, the GDP growth and the inflation rates are contaminated each one with only five jumps due to benchmark revisions. We eliminate each jump by substituting the jumping value of the corresponding variable by the average value between the values of the variable immediately before and after the jump.
of the parameters with the empirical likelihood of the data. In a second step, we perform the
Metropolis-Hastings algorithm to compute the posterior distribution of the parameter set.\(^{13}\)

In terms of the priors, we select the same prior distributions as SW for the estimation of the
model (see the first three columns in Tables 1A and 1B). We have also borrowed their notation for
the structural parameters.\(^{14}\)

4 Estimation results

Tables 1A, 1B and 1C show the estimation results obtained both using real time and revised
data together (i.e. extended model) and only revised data (i.e. SW model). More precise, these
tables report the posterior mean estimates together with the 5\% and 95\% quantiles of the posterior
distribution for the parameters of the two models.

Before we analyze the estimation results, we start by discussing the goodness of the estimation.
Dynare package supplies, as a by-product, several tests such as graphical convergence diagnostic
tests suggested by Brooks and Gelman (1998), which are not shown to save space. According
to these graphical tests, the overall performance is good. Another way to analyze the quality
of estimation results is carried out by comparing the prior and posterior distributions for each
parameter as displayed in Figure 1. In general, we can conclude that estimation results show
that the data are informative about the posterior distribution of the parameters. There are two
exceptions though: Frisch elasticity parameter, \(\sigma_l\), and the output gap coefficient in the policy
rule, \(r_y\). Finally, as another test of the goodness of estimation, the smoothed estimates of the shock
innovation paths displayed in Figure 2 show that these innovation estimates look clearly stationary.

\(\text{\textsuperscript{13}}\)All estimation exercises are performed with DYNARE free routine software, which can be downloaded from
http://www.dynare.org. A sample of 250,000 draws was used (ignoring the first 20\% of draws). A step size of 0.3
resulted in an average acceptance rate of roughly 25\% across the five Metropolis-Hastings blocks used.

\(\text{\textsuperscript{14}}\)See also Tables A.1 and A.2 for a description of model parameters.
As the last three columns of Tables 1A and 1B show, our version of the SW model -with a four-
year longer sample period- confirms SW estimates of the structural parameters. In particular, the
confidence band for each structural parameter -displayed in Table 1A- overlaps to a great extent with
the corresponding confidence interval reported by SW. A similar conclusion regarding the estimated
parameters of the shock processes -displayed in Table 1B- is reached with two exceptions. The
standard deviation of government spending and policy rule shocks are slightly, but significantly,
smaller in our sample whereas our persistence estimate of the policy shock is larger than those
reported in SW.

We now discuss the estimates of the augmented NK model with revision processes by considering
revised and real-time data. We start discussing the parameter estimates associated with revision
processes shown in Table 1C. $b_{yy}$ is the only significant coefficients among the $b_{ij}$ revision process
coefficients. The persistence parameters associated with the two shock revision processes are both
significant, but $\rho_y (= 0.90)$ is rather large whereas $\rho_\pi (= 0.12)$ is quite small. Moreover, the
standard deviation of the output revision innovation, $\sigma_y$, is more than twice larger than the one
associated with inflation revision innovation, $\sigma_\pi$. In sum, the revision process estimates suggest
that the revisions of output and inflation are not rational revision errors in line with the empirical
evidence reported by Aruoba (2008) since, on the one hand, real time data on output helps to
forecast future output data revisions. On the other hand, the two revision process shocks show
significant persistence, although persistence and shock size are much larger in the output than in
the inflation revision process. A careless reading of these estimation results might interpret that the
finding of most $b_{ij}$’s being non-significant somewhat challenges Arouba’s (2008) empirical evidence
about the ability of the initial announcements to forecast revisions of output growth and inflation.
On the contrary, since revision process shocks are persistent state-variables driving agents decisions, both revisions and initial announcements of output growth and inflation are in general determined by these shocks, which implies that the initial announcements help to predict future revisions of output growth and inflation due to shock revision persistence.\textsuperscript{15}

Comparing the set of estimates obtained from the two models -first three versus last three columns of posterior estimates in Tables 1A and 1B- a clear conclusion emerges: most structural, policy and shock process parameter estimates do not change significantly by considering real-time data in addition to revised data. Put differently, the structural parameter estimates of medium-scale NK models are not affected when accounting for the presence of revisions in output and inflation data.

In spite of this general robustness result, it is worth to pointing out two important differences. First, price and wage Calvo probabilities ($\xi_p$ and $\xi_w$) are larger and price and wage indexation parameter estimates ($\iota_p$ and $\iota_w$) are slightly smaller with the extended model and the extended set of observable variables. Second, policy rule inflation coefficient, $r_\pi$, is smaller when considering both revised and real-time data than when using only revised data, but it is significantly larger than one for any standard significance level, supporting Taylor principle. This estimation result contrast with Orphanides’ (2001) result that Taylor rule does not hold when considering real-time data.

\[\text{[Insert Figure 3 and Figure 4]}\]

Figures 3-6 show a selected set of impulse-response functions.\textsuperscript{16} Figures 3 and 4 show the\\textsuperscript{15}Put differently, although initial announcements and final revisions are released in different time periods both are determined by the same set of minimum-state variables. So they are in general correlated with each other as long as the revision process shocks are persistent.\\textsuperscript{16}The full set of impulse-response functions are available from the author upon request.
impulse-responses for a policy rule shock for the extended and SW models. A comparison of these two figures shows that the response of all variables to a policy shock are qualitatively similar in the two models. For most variables such as output, consumption and investment growth rates, the responses are larger in the extended model than in the SW model. However, the opposite occurs for the response of inflation (both when analyzed in deviations from the steady state -denoted by \( p_i \) in the graph- and in levels -denoted by \( p_{iobs} \)).

[Insert Figure 5 and Figure 6]

Figure 5 shows the impulse-responses to a positive output revision shock. This shock increases output revision, which increases revised output, consumption, labor, capital stock and investment. Moreover, the increase of output revision reduces the initial announcement of output, but this reduction barely affects nominal interest rate due to the small coefficient associated with output gap in the policy rule.

Finally, Figure 6 shows the impulse-responses to a positive inflation revision shock. This shock increases inflation revision, which reduces the initial announcement of inflation leading to a reduction of the nominal interest, which results in larger revised output, consumption, investment, capital and employment.

[Insert Table 2]

Table 2 shows the total variance decomposition analysis for the extended and SW models. In the extended model, the two revision innovations, \( \eta^y \) and \( \eta^\pi \), have a weak impact in all variables but the initial announcements of output growth rate (32.6%) and inflation (42.2%), respectively. A comparison of the two panels in Table 2, show that by considering real time data the relative
importance of wage-push, $\eta^w$, and interest rate, $\eta^R$, innovations change substantially. More precise, the impact of the growth rates of output, consumption, investment and real wage to a wage-push innovation are significantly weaker in the extended model than in the SW model. The opposite occurs with the impact of real wage growth rate to a wage-push innovation. Moreover, the interest rate innovation is significantly more influential for hours worked, inflation and nominal interest rate in the extended model than in the SW model. The relative importance of the remaining innovations -technology, risk premium, fiscal/net and investment adjustment cost- is basically not affected by taking into account real time data.

5 Conclusions
References


Table 1A. Priors and estimated posteriors of the structural parameters

<table>
<thead>
<tr>
<th></th>
<th>Priors</th>
<th>Postiors</th>
<th>Real time &amp; Revised</th>
<th>Only Revised</th>
</tr>
</thead>
<tbody>
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<td>Distr</td>
<td>Mean</td>
<td>Std D.</td>
<td>Mean</td>
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<td>$h$</td>
<td>Beta</td>
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<td>0.76</td>
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<td>0.12</td>
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<td>$r_\pi$</td>
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<td>$\rho$</td>
<td>Beta</td>
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<td>Gamma</td>
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<td>0.10</td>
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<td>$100(\beta^{-1}-1)$</td>
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Table 1B. Priors and estimated posteriors of the shock processes

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<td>Invgamma</td>
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Table 1C. Priors and estimated posteriors of revision processes parameters

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<td>95%</td>
<td>Mean 5% 95%</td>
<td>Mean 5% 95%</td>
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<td>2.00</td>
<td>0.07 −0.25 0.36</td>
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<tr>
<td>$b_{πy}$</td>
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<td>0.00 −0.01 0.02</td>
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Table 2. Variance decomposition (percent)

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<th>SW model</th>
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<td>$\Delta y$</td>
<td>$\Delta y^f$</td>
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<tr>
<td>Technology, $\eta^a$</td>
<td>3.4</td>
<td>2.3</td>
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<td>Risk premium, $\eta^b$</td>
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<td>Fiscal/Net exports, $\eta^g$</td>
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<td>Price-push, $\eta^p$</td>
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<tr>
<td>Inflation revision, $\eta^\pi$</td>
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Table 3. Second-moment statistics

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<th>$\Delta c$</th>
<th>$\Delta i$</th>
<th>$\Delta w$</th>
<th>$l$</th>
<th>$R$</th>
<th>$\pi$</th>
<th>$\Delta y^r$</th>
<th>$\pi^r$</th>
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<td>0.73</td>
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<td>0.72</td>
<td>0.51</td>
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<tr>
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<td>0.63</td>
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<td>0.51</td>
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<tr>
<td>Standard deviation (%)</td>
<td>0.95</td>
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<td>2.27</td>
<td>0.92</td>
<td>3.02</td>
<td>0.50</td>
<td>0.36</td>
<td>0.93</td>
<td>0.49</td>
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<td>Correlation with output growth</td>
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<td>0.78</td>
<td>0.72</td>
<td>0.24</td>
<td>0.15</td>
<td>-0.35</td>
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<td>0.94</td>
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<tr>
<td>Standard deviation (%)</td>
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<td>0.91</td>
<td>3.83</td>
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<td>Correlation with output growth</td>
<td>1.0</td>
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<td>0.13</td>
<td>-0.35</td>
<td>-0.23</td>
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<td>0.98</td>
<td>0.95</td>
<td>0.82</td>
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</tbody>
</table>
Appendix

Set of log-linearized dynamic equations:

- Inflation identity:
  \[ \pi_t = \pi^r_t + r^\pi_t. \]  
  \hspace{1cm} (A1)

- Output identity:
  \[ y_t \equiv y^r_t + r^y_t. \]  
  \hspace{1cm} (A2)

- Revision process of inflation:
  \[ r^\pi_t = b_{\pi y} y^r_t + b_{\pi \pi} \pi^r_t + \varepsilon^\pi_t. \]  
  \hspace{1cm} (A3)

- Revision process of output:
  \[ r^y_t = b_{y y} y^r_t + b_{y \pi} \pi^r_t + \varepsilon^y_t. \]  
  \hspace{1cm} (A4)

- Aggregate resource constraint:
  \[ y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon^y_t, \]  
  \hspace{1cm} (A5)

where \( c_y = \frac{C}{Y} = 1 - g_y - i_y, \) \( i_y = \frac{I}{Y} = (\gamma - 1 + \delta) \frac{K}{Y}, \) and \( z_y = r^K \frac{K}{Y} \) are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at \( \delta = 0.025 \) and \( g_y = 0.18. \)

- Consumption equation:
  \[ c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (R_t - E_t \pi_{t+1}) + \varepsilon^b_t, \]  
  \hspace{1cm} (A6)

where \( c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}, \) \( c_2 = \frac{[(\sigma_c - 1)wL/(\phi_w C)]}{\sigma_c(1+\lambda/\gamma)} \) and \( c_3 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}. \)

- Investment equation:
  \[ i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon^i_t, \]  
  \hspace{1cm} (A7)

where \( i_1 = \frac{1}{1+\beta}, \) and \( i_2 = \frac{1}{(1+\beta)^2\gamma^2} \) with \( \beta = \beta\gamma^{(1-\sigma_c)}. \)
• Arbitrage condition (value of capital, $q_t$):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} e_t^b,$$  \hfill (A8)

where $q_1 = \beta \gamma^{-1} (1 - \delta) = \frac{(1-\delta)}{(r^* + 1 - \delta)}$.

• Log-linearized aggregate production function:

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^q),$$  \hfill (A9)

where $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and $\alpha$ is the capital-share in the production function.\footnote{From the zero profit condition in steady-state, it should be noticed that $\phi_p$ also represents the value of the steady-state price mark-up.}

• Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t.$$  \hfill (A10)

• Capital utilization:

$$z_t = z_1 r_t^k,$$  \hfill (A11)

where $z_1 = \frac{1 - \psi}{\psi}$.

• Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i,$$  \hfill (A12)

where $k_1 = \frac{1 - \delta}{\gamma}$ and $k_2 = \left(1 - \frac{1 - \delta}{\gamma}\right) \left(1 + \beta\right) \gamma^2 \phi$.

• Price mark-up (negative of the log of the real marginal cost):

$$\mu_t^p = m p_t - w_t = \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t.$$  \hfill (A13)
• New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \pi_t^p + \overline{\beta} \pi_1 E_t \pi_t^x + \varepsilon_t^p,$$

(A14)

where \( \pi_1 = \frac{\varepsilon_p}{1+3\beta_p}, \ \pi_2 = \frac{\overline{\beta}}{1+3\beta_p}, \ \text{and} \ \pi_3 = \frac{1}{1+3\beta_p} \left[ \frac{(1-\overline{\eta}_p)(1-\varepsilon_p)}{\eta_p((\phi_p-1)\varepsilon_p+1)} \right]. \) The coefficient of the curvature of the Kimball goods market aggregator is fixed in the estimation procedure at \( \varepsilon_p = 10 \) as in Smets and Wouters (2007).

• Optimal demand for capital by firms:

$$-(k_t^s - l_t) + w_t = \frac{1}{pk_t^k}.$$

(A15)

• Wage markup equation:

$$\mu_t^w = w_t - \text{mrs}_t = w_t - \left( \sigma_t l_t + \frac{1}{1-\gamma} (c_t - \lambda/\gamma c_{t-1}) \right).$$

(A16)

• Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + w_5 \beta w E_t \pi_t^x + \varepsilon_t^w,$$

(A17)

where \( w_1 = \frac{1}{1+\beta}, \ w_2 = \frac{1+\overline{\beta}}{1+\beta}, \ w_3 = \frac{\overline{\beta}}{1+\beta}, \ w_4 = \frac{1}{1+\beta} \left( \frac{(1-\overline{\eta}_w)(1-\varepsilon_w)}{\eta_w((\phi_w-1)\varepsilon_w+1)} \right) \) with the curvature of the Kimball labor aggregator fixed at \( \varepsilon_w = 10 \) and a steady-state wage mark-up fixed at \( \phi_w = 1.5 \) as in Smets and Wouters (2007).

• Monetary policy rule, a Taylor-type rule for nominal interest rate management:

$$R_t = \rho R_{t-1} + (1-\rho) \left[ r_x (\pi_{t-1}^x + E_{t-2} \pi_{t-1}^x) + r_y (y_{t-1}^x + E_{t-2} y_{t-1}^x - y_{t-1}^p) \right] + \varepsilon_t^R.$$

(A18)

Potential (natural-rate) variables, assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks.

• Flexible-price condition (no price mark-up fluctuations, \( \mu_t^p = m p l_t = w_t = 0 \)):

$$\alpha (k_t^{s,p} - l_t^{p}) + \varepsilon_t^a = w_t^p.$$

(A19)
• Flexible-wage condition (no wage mark-up fluctuations, \( \mu^w_t = w_t - m_r s_t = 0 \)):

\[
w^p_t = \sigma I^p_t + \frac{1}{1-\gamma/\gamma} (c^p_t - \lambda/c^p_{t-1}) .
\]  

(A20)

• Potential aggregate resources constraint:

\[
y^p_t = c y^p_t + i p^p_t + z_q^p + \varepsilon^p .
\]  

(A21)

• Potential consumption equation:

\[
c^p_t = c_1 c^p_{t-1} + (1 - c_1) E_t c^p_{t+1} + c_2 (p^p_t - E_t p^p_{t+1}) - c_3 (R^p_t - E_t \pi^p_{t+1}) + \varepsilon^p .
\]  

(A22)

• Potential investment equation:

\[
i^p_t = i_1 i^p_{t-1} + (1 - i_1) E_t i^p_{t+1} + i_2 q^p + \varepsilon^i .
\]  

(A23)

• Arbitrage condition (value of potential capital, \( q^p_t \)):

\[
q^p_t = q_1 E_t q^p_{t+1} + (1 - q_1) E_t k^p_{t+1} - (R^p_t - E_t \pi^p_{t+1}) + c_3^{-1} \varepsilon^b .
\]  

(A24)

• Log-linearized potential aggregate production function:

\[
y^p_t = \phi_p (\alpha k^s,p_t + (1 - \alpha) l^p_t + \varepsilon^p) .
\]  

(A25)

• Potential capital (with one period time-to-build):

\[
k^s,p_t = k^p_{t-1} + z^p_t .
\]  

(A26)

• Potential capital utilization:

\[
z^p_t = z_1 r^k,p_t .
\]  

(A27)

• Potential capital accumulation equation:

\[
k^p_t = k_1 k^p_{t-1} + (1 - k_1) i^p_t + k_2 \varepsilon^i .
\]  

(A28)
• Potential demand for capital by firms ($r^{k,p}_t$ is the potential log of the rental rate of capital):

$$-(k^{s,p}_t - l^p_t) + w^p_t = \frac{1}{\rho^{k,p}_t} r^{k,p}_t. \quad (A29)$$

• Monetary policy rule (under flexible prices and flexible wages):

$$R^p_t = \rho R^p_{t-1} + (1 - \rho) [r_{\pi} \pi^p_t] + \varepsilon^R_t. \quad (A30)$$

**Equations-and-variables summary**

- Set of equations:

  Equations (A1)-(A30) determine solution paths for 18 endogenous variables.

- Set of variables:

  **Endogenous variables (30):** $y_t, c_t, i_t, z_t, l_t, R_t, \pi_t, q_t, r^k_t, k_t, k^s_t, k^p_t, \mu^w_t, \mu^p_t, w_t, y^r_t, \pi^r_t, r^p_t, y^p_t, c^p_t, i^p_t, z^p_t, l^p_t, R^p_t, \pi^p_t, q^p_t, r^{k,p}_t, k^{s,p}_t, k^p_t, \text{and } w^p_t$.

  **Predetermined variables (15):** $c_{t-1}, i_{t-1}, k_{t-1}, \pi_{t-1}, w_{t-1}, R_{t-1}, y_{t-1}, y^r_{t-1}, \pi^r_{t-1}, r^p_{t-1}, r^\pi_{t-1}, c^p_{t-1}, i^p_{t-1}, k^p_{t-1}, \text{and } r^p_{t-1}$.

  **Exogenous variables (9):** AR(1) technology shock $\varepsilon^a_t = \rho_a \varepsilon^a_{t-1} + \eta^a_t$, AR(1) risk premium shock $\varepsilon^b_t = \rho_b \varepsilon^b_{t-1} + \eta^b_t$, AR(1) exogenous spending shock cross-correlated to technology innovations $\varepsilon^g_t = \rho_g \varepsilon^g_{t-1} + \eta^g_t$, AR(1) investment shock $\varepsilon^i_t = \rho_i \varepsilon^i_{t-1} + \eta^i_t$, AR(1) monetary policy shock $\varepsilon^R_t = \rho_R \varepsilon^R_{t-1} + \eta^R_t$, ARMA(1,1) price mark-up shock $\varepsilon^p_t = \rho_p \varepsilon^p_{t-1} + \eta^p_t - \mu_p \eta^p_{t-1}$, ARMA(1,1) wage mark-up shock $\varepsilon^w_t = \rho_w \varepsilon^w_{t-1} + \eta^w_t - \mu_w \eta^w_{t-1}$, AR(1) output revision shock $\varepsilon^\pi_t = \rho^\pi \varepsilon^{\pi}_t + \eta^{\pi}_t$ and AR(1) inflation revision shock $\varepsilon^{\pi^r}_t = \rho^{\pi^r} \varepsilon^{\pi^r}_{t-1} + \eta^{\pi^r}_t$. 

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Elasticity of the cost of adjusting capital</td>
</tr>
<tr>
<td>$h$</td>
<td>External habit formation</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Inverse of the elasticity of intertemporal substitution in utility function</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Inverse of the elasticity of labor supply with respect to the real wage</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Calvo probability that measures the degree of price stickiness</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Calvo probability that measures the degree of wage stickiness</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>Degree of wage indexation to past wage inflation</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>Degree of price indexation to past price inflation</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of capital utilization adjustment cost</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>(One plus) steady-state fixed cost to total cost ratio. Price mark-up</td>
</tr>
<tr>
<td>$r_\pi$</td>
<td>Inflation coefficient in monetary policy rule</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Policy inertia parameter</td>
</tr>
<tr>
<td>$r_\gamma$</td>
<td>Output gap coefficient in monetary policy rule</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady-state rate of inflation</td>
</tr>
<tr>
<td>$100(\beta^{-1}-1)$</td>
<td>Steady-state rate of discount</td>
</tr>
<tr>
<td>$l$</td>
<td>Steady-state rate of labor growth</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Steady-state growth rate</td>
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<tr>
<td>$\alpha$</td>
<td>Capital share in production function</td>
</tr>
<tr>
<td>$b_{yy}$</td>
<td>Output coefficient in output revision process</td>
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<tr>
<td>$b_{y\pi}$</td>
<td>Inflation coefficient in output revision process</td>
</tr>
<tr>
<td>$b_{\pi y}$</td>
<td>Output coefficient in inflation revision process</td>
</tr>
<tr>
<td>$b_{\pi \pi}$</td>
<td>Inflation coefficient in inflation revision process</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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</tr>
<tr>
<td>$\sigma_a$</td>
<td>Standard deviation of productivity innovation</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Standard deviation of risk premium innovation</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>Standard deviation of exogenous spending innovation</td>
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<tr>
<td>$\sigma_i$</td>
<td>Standard deviation of investment-specific innovation</td>
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<tr>
<td>$\sigma_R$</td>
<td>Standard deviation of policy rule innovation</td>
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<td>Standard deviation of price mark-up innovation</td>
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<td>$\sigma_w$</td>
<td>Standard deviation of wage mark-up innovation</td>
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<td>Standard deviation of output gap revision process innovation</td>
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<td>$\sigma_{\pi}^r$</td>
<td>Standard deviation of inflation revision process innovation</td>
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<tr>
<td>$\rho_a$</td>
<td>Autoregressive coefficient of productivity shock process</td>
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<td>$\rho_b$</td>
<td>Autoregressive coefficient of risk premium shock process</td>
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<td>Autoregressive coefficient of price mark-up shock process</td>
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<td>Autoregressive coefficient of wage mark-up shock process</td>
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<tr>
<td>$\mu_p$</td>
<td>Moving-average coefficient of price mark-up shock process</td>
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<td>Moving-average coefficient of wage mark-up shock process</td>
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<td>$\rho_{ga}$</td>
<td>Productivity innovation coefficient of exogenous spending shock process</td>
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<td>$\rho_{yr}$</td>
<td>Autoregressive coefficient of output revision shock process</td>
</tr>
<tr>
<td>$\rho_{\pi r}$</td>
<td>Autoregressive coefficient of inflation revision shock process</td>
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Figure 1.A: Prior and posterior distributions of structural parameters
Figure 1.B: Prior and posterior distributions of the structural parameters (continued)
Figure 1.C: Prior and posterior distributions of the structural parameters (continued)

Figure 2: Smoothed estimates of innovations
Figure 3: Impulse response functions to a monetary policy shock (extended model)
Figure 4: Impulse response functions to a monetary policy shock (SW model)
Figure 5: Impulse response functions to an output revision shock
Figure 6: Impulse response functions to an inflation revision shock